CBSE Class 12 Mathematics: Sample Paper 1: Marking Scheme and Solutions

Based on NCERT · the board's 2026-27 pattern | Time: 3 hours · **Maximum marks: 80**

Section A (20 × 1 mark)

1. The vector $12\hat{i} + y\hat{j} - 4\hat{k}$, with $y > 0$, has magnitude 13. The value of y is [1]

(a) 5 (b) 9

(c) 1 (d) 3

Answer: (d) $144 + y^2 + 16 = 169$, so $y^2 = 9$ and, as $y > 0$, $y = 3$. Stopping at y^2 gives 9.

2. In a linear programming problem with decision variables x and y, which of the following is a non-negative restriction? [1]

(a) $x + y \le 12$ (b) $x - 2y \ge 0$

(c) $Z = 5x + 3y$ (d) $y \ge 0$

Answer: (d) The conditions $x \ge 0$, $y \ge 0$ are the non-negative restrictions. $x + y \le 12$ and $x - 2y \ge 0$ are other constraints ($x - 2y \ge 0$ allows negative values of y), and $Z = 5x + 3y$ is an objective function.

3. $\int_1^3 (2x + 1)\,dx$ equals [1]

(a) 9 (b) 8

(c) 12 (d) 10

Answer: (d) An anti derivative is $F(x) = x^2 + x$. $F(3) - F(1) = 12 - 2 = 10$.

4. For a value of x at which $\sin x \ne 0$, the expression $(\sin x)^{-1}$ means [1]

(a) $-\sin x$ (b) $\sin^{-1} x$

(c) cosec x (d) $\sin(-x)$

Answer: (c) $(\sin x)^{-1}$ is the reciprocal $1/\sin x$ = cosec x. It must not be confused with $\sin^{-1} x$, the inverse function.

5. Let R be the feasible region of a linear programming problem and $Z = ax + by$ the objective function. If R is bounded, then [1]

(a) Z has a maximum value on R but may have no minimum value

(b) the maximum of Z occurs only at a point inside R, not on its boundary

(c) Z may have neither a maximum nor a minimum value on R

(d) Z has both a maximum and a minimum value on R, each occurring at a corner point of R

Answer: (d) Theorem 2: if R is bounded, Z has both a maximum and a minimum value on R and each of these occurs at a corner point (vertex) of R. The possibility of no maximum or no minimum arises only for an unbounded region.

6. The principal value branch of which of the following functions has the open interval $(0, \pi)$ as its range? [1]

(a) $\cos^{-1}$ (b) $\cot^{-1}$

(c) $\sec^{-1}$ (d) $\tan^{-1}$

Answer: (b) $\cot^{-1} : R \to (0, \pi)$. $\cos^{-1}$ has the closed range $[0, \pi]$; $\sec^{-1}$ has $[0, \pi] - \{\pi/2\}$, which contains 0 and π; $\tan^{-1}$ has $(-\pi/2, \pi/2)$.

7. If $\begin{pmatrix} x+y & 3 \\ 2 & x-y \end{pmatrix} = \begin{pmatrix} 9 & 3 \\ 2 & 1 \end{pmatrix}$, then xy equals [1]

(a) 1 (b) 9

(c) 20 (d) 16

Answer: (c) Equal matrices have equal corresponding elements: $x + y = 9$ and $x - y = 1$, so $x = 5$, $y = 4$ and $xy = 20$.

8. The area of the region bounded by $y = |x - 2|$, the x-axis and the ordinates $x = 0$ and $x = k$, where $k > 2$, is 10 square units. Then k equals [1]

(a) 6 (b) 4

(c) $2 + 2\sqrt{5}$ (d) 10

Answer: (a) Split at $x = 2$. On $[0, 2]$ the region is a triangle of area $\frac{2^2}{2} = 2$; on $[2, k]$ it is a triangle of area $\frac{(k-2)^2}{2}$. So $2 + \frac{(k-2)^2}{2} = 10$, $(k - 2)^2 = 16$ and $k = 6$. $2 + 2\sqrt{5}$ ignores the triangle on $[0, 2]$.

9. The degree of the differential equation $(y'')^2 + (y')^3 - 7y = 0$ is [1]

(a) 1 (b) 3

(c) 2 (d) 5

Answer: (c) The equation is a polynomial in y' and y''. The highest order derivative is y'', and its highest power is 2, so the degree is 2. Option (b) takes the highest power of any derivative; (d) adds the two powers.

10. If $(p - 2)\hat{i} + 3\hat{j} - q\hat{k} = 5\hat{i} + (r + 1)\hat{j} + 4\hat{k}$, then $p + q + r$ equals [1]

(a) 9 (b) 13

(c) 5 (d) 1

Answer: (c) Equal vectors have equal components: $p - 2 = 5$, $3 = r + 1$, $-q = 4$. So $p = 7$, $r = 2$, $q = -4$ and $p + q + r = 5$. Taking $q = 4$ gives 13.

11. A and B are mutually exclusive events with $P(A) = 0.3$ and $P(B) = 0.4$. Which of the following is true? [1]

(a) A and B are independent (b) $P(A \cap B) = 0.12$

(c) $P(A|B) = 0$ (d) $P(A \cup B) = 0.58$

Answer: (c) Mutually exclusive means $A \cap B = \phi$, so $P(A \cap B) = 0$ and $P(A|B) = 0/0.4 = 0$. Since $P(A)$ $P(B) = 0.12 \neq 0$, A and B are not independent, which rules out (a) and (b); $P(A \cup B) = 0.3 + 0.4 = 0.7$, not 0.58, which would be the value for independent events.

12. The number of relations in the set {1, 2, 3} that are both reflexive and symmetric is [1]

(a) 6 (b) 64

(c) 8 (d) 512

Answer: (c) The three pairs (1, 1), (2, 2), (3, 3) must be present. Symmetry makes the other six pairs come in three couples {(1, 2), (2, 1)}, {(1, 3), (3, 1)}, {(2, 3), (3, 2)}, each wholly in or wholly out: $2^3 = 8$. 64 counts reflexive relations without the symmetry condition.

13. The line joining the origin O to the point $P(1, 1, \sqrt{2})$ makes with the positive direction of the z-axis an angle of [1]

(a) 60° (b) 45°

(c) 30° (d) 90°

Answer: (b) $OP = \sqrt{(1 + 1 + 2)} = 2$, so $\cos\gamma = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$ and $\gamma = 45°$. Option (a) takes $\cos\gamma = \frac{1}{2}$ from the x or y coordinate.

14. The function $f(x) = \frac{x^3 - 8}{x - 2}$ for $x \neq 2$, and $f(2) = k$, is continuous at $x = 2$. The value of k is [1]

(a) 4 (b) 6
(c) 12 (d) 0

Answer: (c) For $x \neq 2$, $(x^3 - 8)/(x - 2) = x^2 + 2x + 4$, so $\lim_{x\to 2} f(x) = 4 + 4 + 4 = 12$. Continuity at 2 needs $f(2) = 12$, so $k = 12$.

15. A jar at a school fair holds n coupons, exactly 2 of which win a prize. Two coupons are drawn one after the other without replacement. If the probability that both coupons win is $\frac{1}{15}$, then n is [1]

(a) 5 (b) 15
(c) 6 (d) 30

Answer: (c) P(first wins) = 2/n and P(second wins | first wins) = 1/(n − 1), so $2/(n(n - 1)) = 1/15$, $n(n - 1) = 30$ and $n = 6$. Check: 2/6 × 1/5 = 1/15. Option (a) gives 1/10; (b) treats the draws as one draw of a winner.

16. A and B are independent events with $P(A) = 0.3$ and $P(A \cap B) = 0.12$. Then $P(B')$ is [1]

(a) 0.6 (b) 0.4
(c) 0.88 (d) 0.036

Answer: (a) For independent events $P(A \cap B) = P(A)\,P(B)$, so $P(B) = 0.12/0.3 = 0.4$ and $P(B') = 1 - 0.4 = 0.6$. Option (b) is P(B); (c) is $1 - P(A \cap B)$.

17. The critical points of $f(x) = x^3 - 27x$ are [1]

(a) $x = 3$ only (b) $x = -3$ and $x = 3$
(c) $x = 0$ and $x = 3$ (d) $x = 0$ only

Answer: (b) $f'(x) = 3x^2 - 27 = 3(x - 3)(x + 3) = 0$ at $x = \pm 3$; f is differentiable everywhere, so these are the only critical points.

18. Two lines in space that are neither parallel nor intersecting are called [1]

(a) coplanar lines (b) skew lines
(c) concurrent lines (d) perpendicular lines

Answer: (b) Lines in space which are neither parallel nor intersecting are skew lines; they lie in different planes, so (a) is wrong. Perpendicular lines may intersect, and concurrent lines meet.

19. Assertion (A): The area of the region bounded by $y = -x^2$, the x-axis and the ordinates $x = 0$ and $x = 3$ is 9 square units. Reason (R): If $f(x) \leq 0$ on $[a, b]$, the area bounded by $y = f(x)$, the x-axis and the ordinates $x = a$ and $x = b$ is $\left|\int_a^b f(x)\,dx\right|$. [1]

(a) Both A and R are true, and R is the correct explanation of A. (b) Both A and R are true, but R is not the correct explanation of A.
(c) A is true, but R is false. (d) A is false, but R is true.

Answer: (a) $\int_0^3 (-x^2)\,dx = -9$; the curve is below the x-axis, so the area is $|-9| = 9$. R is the rule that gives A.

20. Assertion (A): The differential equation $(y''')^2 + \log(y') = x$ has order 3, and its degree is not defined. Reason (R): The degree of a differential equation is defined only when it is a polynomial equation in its derivatives. [1]

(a) Both A and R are true, and R is the correct explanation of A.
(b) Both A and R are true, but R is not the correct explanation of A.
(c) A is true, but R is false.
(d) A is false, but R is true.

Answer: (a) The highest derivative is y''', so the order is 3. $\log(y')$ is not a polynomial in y', so by R the degree is not defined. R is exactly the reason for the second half of A.

Section B (5 × 2 marks)

21. Find the general solution of the differential equation $3y^2 \frac{dy}{dx} = \sec^2 x$. [2]

Or *Find the general solution of the differential equation* $\frac{dy}{dx} = x\,e^{-2y}$.

Separating the variables, $3y^2\,dy = \sec^2 x\,dx$. Integrating, $\int 3y^2\,dy = \int \sec^2 x\,dx$, so $y^3 = \tan x + C$, the general solution.

- separates: $3y^2\,dy = \sec^2 x\,dx$ (1)
- $y^3 = \tan x + C$ (1)

Or:

Separating, $e^{2y}\,dy = x\,dx$. Integrating, $e^{2y}/2 = x^2/2 + C/2$, so $e^{2y} = x^2 + C$, the general solution.

- separates: $e^{2y}\,dy = x\,dx$ (1)
- $e^{2y} = x^2 + C$ (or an equivalent form) (1)

22. Let $f : R \to R$, $f(x) = x^2 - 4x$, and let $R = \{(a, b) : f(a) = f(b)\}$ in R, an equivalence relation. Find the equivalence class [1]. [2]

Or *Let* $g : Z \to Z$, $g(x) = |x - 2|$, *and let* $R = \{(a, b) : g(a) = g(b)\}$ *in Z, an equivalence relation. Find the equivalence class [5].*

$[1] = \{b \in R : f(b) = f(1)\}$ and $f(1) = 1 - 4 = -3$. So $b^2 - 4b = -3$, $b^2 - 4b + 3 = 0$, $(b - 1)(b - 3) = 0$, $b = 1$ or 3. Hence $[1] = \{1, 3\}$.

- sets up $f(b) = f(1) = -3$ (1)
- $[1] = \{1, 3\}$ (1)

Or:

$[5] = \{b \in Z : |b - 2| = |5 - 2| = 3\}$. So $b - 2 = 3$ or $b - 2 = -3$, $b = 5$ or $b = -1$. Hence $[5] = \{-1, 5\}$.

- sets up $|b - 2| = 3$ (1)
- $[5] = \{-1, 5\}$ (1)

23. For events A and B, $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cup B) = 0.7$. Find $P(A'|B')$. [2]

$P(A' \cap B') = P((A \cup B)') = 1 - 0.7 = 0.3$ and $P(B') = 1 - 0.4 = 0.6$. So $P(A'|B') = P(A' \cap B')/P(B') = 0.3/0.6 = 0.5$.

- $P(A' \cap B') = 1 - P(A \cup B) = 0.3$ (1)
- $P(B') = 0.6$ and $P(A'|B') = 0.5$ (1)

24. A line through the origin O makes angles of 120° and 60° with the positive directions of the x-axis and the y-axis, and an acute angle with the positive direction of the z-axis. Find the coordinates of the point P on this line, on the side of that acute angle, with OP = 4 units. [2]

$\cos^2\gamma = 1 - \cos^2 120° - \cos^2 60° = 1 - \frac{1}{4} - \frac{1}{4} = \frac{1}{2}$; γ is acute, so $\cos\gamma = \frac{1}{\sqrt{2}}$. The direction cosines are $-\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{\sqrt{2}}$, and $P = 4(l, m, n) = (-2, 2, 2\sqrt{2})$.

- $\cos\gamma = \frac{1}{\sqrt{2}}$ from $l^2 + m^2 + n^2 = 1$ (1)
- $P = (-2, 2, 2\sqrt{2})$ (1)

25. Find the values of x for which $\begin{vmatrix} x & 6 \\ 2 & x \end{vmatrix} = \begin{vmatrix} 4 & 5 \\ -1 & 2 \end{vmatrix}$. [2]

Left side: $x^2 - 12$. Right side: $8 - (-5) = 13$. So $x^2 - 12 = 13$, $x^2 = 25$, $x = \pm 5$.

- both determinants expanded: $x^2 - 12$ and 13 (1)
- $x^2 = 25$, $x = \pm 5$ (1)

Section C (6 × 3 marks)

26. Find the value of $\sin(2\tan^{-1}(1/3)) + \cos(\tan^{-1}(2\sqrt{2}))$. [3]

Or *Find the value of $\sin(2\tan^{-1}(1/4)) - \cos(\tan^{-1}\sqrt{15})$.*

Let $\alpha = \tan^{-1}(1/3)$. $\sin 2\alpha = 2\tan\alpha/(1 + \tan^2\alpha) = (2/3)/(10/9) = 3/5$. Let $\beta = \tan^{-1}(2\sqrt{2}) \in (0, \pi/2)$, so $\tan\beta = 2\sqrt{2}$, $\sec^2\beta = 1 + 8 = 9$ and $\cos\beta = 1/3$ (positive in $(0, \pi/2)$). Sum $= 3/5 + 1/3 = 14/15$.

- $\sin(2\tan^{-1}(1/3)) = 3/5$ (1)
- $\cos(\tan^{-1} 2\sqrt{2}) = 1/3$ with the sign justified (1)
- sum 14/15 (1)

Or:

Let $\alpha = \tan^{-1}(1/4)$. $\sin 2\alpha = 2\tan\alpha/(1 + \tan^2\alpha) = (1/2)/(17/16) = 8/17$. Let $\beta = \tan^{-1}\sqrt{15} \in (0, \pi/2)$, so $\sec^2\beta = 1 + 15 = 16$ and $\cos\beta = 1/4$ (positive in $(0, \pi/2)$). Value $= 8/17 - 1/4 = (32 - 17)/68 = 15/68$.

- $\sin(2\tan^{-1}(1/4)) = 8/17$ (1)
- $\cos(\tan^{-1}\sqrt{15}) = 1/4$ with the sign justified (1)
- value 15/68 (1)

27. Maximise $Z = 4x + 3y$ subject to $x + y \le 8$, $x \le 5$, $y \le 6$, $x \ge 0$, $y \ge 0$, by the corner point method. By how much would the maximum value increase if the constraint $x \le 5$ were removed? [3]

Or *Maximise $Z = 2x + 5y$ subject to $x + y \le 9$, $x \le 7$, $y \le 6$, $x \ge 0$, $y \ge 0$, by the corner point method. By how much would the maximum value increase if the constraint $y \le 6$ were removed?*

The feasible region is bounded, with corners (0, 0), (5, 0), (5, 3) [x = 5 meets x + y = 8], (2, 6) [y = 6 meets x + y = 8] and (0, 6). Z there: 0, 20, 29, 26, 18; the maximum is 29 at (5, 3). Without $x \le 5$ the corners are (0, 0), (8, 0), (2, 6), (0, 6), with Z = 0, 32, 26, 18: the maximum becomes 32 at (8, 0). The increase is 3.

- corners of the region found (1)
- maximum 29 at (5, 3) (1)
- new maximum 32 at (8, 0); increase 3 (1)

Or:

The feasible region is bounded, with corners (0, 0), (7, 0), (7, 2) [x = 7 meets x + y = 9], (3, 6) [y = 6 meets x + y = 9] and (0, 6). Z there: 0, 14, 24, 36, 30; the maximum is 36 at (3, 6). Without $y \le 6$ the corners are (0, 0), (7, 0), (7, 2), (0, 9), with Z = 0, 14, 24, 45: the maximum becomes 45 at (0, 9). The increase is 9.

- corners of the region found (1)
- maximum 36 at (3, 6) (1)
- new maximum 45 at (0, 9); increase 9 (1)

28. Show that $f(x) = x^2$ for $x \le 1$ and $f(x) = 2x - 1$ for $x > 1$ is differentiable at $x = 1$. [3]

Or *Show that $f(x) = x^2$ for $x \le 1$ and $f(x) = 3x - 2$ for $x > 1$ is continuous but not differentiable at $x = 1$.*

Continuity: f(1) = 1, left hand limit 1, right hand limit 2 − 1 = 1. Left hand derivative: $\lim_{h \to 0^-} [(1 + h)^2 - 1]/h = \lim (2 + h) = 2$. Right hand derivative: $\lim_{h \to 0^+} [2(1 + h) - 1 - 1]/h = 2$. They are finite and equal, so f is differentiable at 1 with $f'(1) = 2$.

- continuity at x = 1 checked (1)
- left hand derivative = 2 (1)
- right hand derivative = 2, so differentiable (1)

Or:

Continuity: f(1) = 1, left hand limit 1, right hand limit 3 − 2 = 1, so f is continuous at 1. Left hand derivative: $\lim_{h \to 0^-} [(1 + h)^2 - 1]/h = 2$. Right hand derivative: $\lim_{h \to 0^+} [3(1 + h) - 2 - 1]/h = 3$. Since $2 \ne 3$, f is not differentiable at 1.

- continuity at x = 1 shown (1)
- left hand derivative = 2 (1)
- right hand derivative = 3, so not differentiable (1)

29. Using vectors, prove that the diagonals of a rhombus are perpendicular to each other. [3]

Let the rhombus be OACB with $\vec{OA} = \vec{a}$ and $\vec{OB} = \vec{b}$, so $|\vec{a}| = |\vec{b}|$. The diagonals are $\vec{OC} = \vec{a} + \vec{b}$ and $\vec{BA} = \vec{a} - \vec{b}$. $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = |\vec{a}|^2 - |\vec{b}|^2 = 0$. Both diagonals are non-zero, so they are perpendicular.

- diagonals written as $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ (1)
- $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = |\vec{a}|^2 - |\vec{b}|^2$ (1)
- $= 0$ as $|\vec{a}| = |\vec{b}|$, hence perpendicular (1)

30. A is a symmetric matrix and B a skew symmetric matrix of the same order. Prove that AB + BA is skew symmetric and AB − BA is symmetric. [3]

$A' = A$ and $B' = -B$. $(AB + BA)' = (AB)' + (BA)' = B'A' + A'B' = (-B)A + A(-B) = -(BA + AB) = -(AB + BA)$, so AB + BA is skew symmetric. $(AB - BA)' = B'A' - A'B' = -BA + AB = AB - BA$, so AB − BA is symmetric.

- uses $A' = A$, $B' = -B$ and $(AB)' = B'A'$ (1)
- $(AB + BA)' = -(AB + BA)$ (1)
- $(AB - BA)' = AB - BA$ (1)

31. Arjun remembers every digit of his phone's PIN except the last one. He tries digits from 0 to 9 at random for the last place, never repeating a digit he has tried. Find the probability that he unlocks the phone within his first three attempts. [3]

Let F_i be 'attempt i fails'. $P(F_1) = 9/10$; given F_1, 9 digits are left, one correct, so $P(F_2|F_1) = 8/9$; $P(F_3|F_1F_2) = 7/8$. By the multiplication rule, P(all three fail) = 9/10 × 8/9 × 7/8 = 7/10. So P(unlocks within three attempts) = 1 − 7/10 = 3/10.

- $P(F_1) = 9/10$, $P(F_2|F_1) = 8/9$, $P(F_3|F_1F_2) = 7/8$ (1)
- product 7/10 by the multiplication rule (1)
- answer 3/10 (1)

Section D (4 × 5 marks)

32. (a) Find where the curve $y = 2\cos x - 1$ meets the x-axis for $0 \le x \le \pi$. (b) Find the area of the region bounded by the curve, the x-axis and the ordinates $x = 0$ and $x = \pi$. (c) Show that $\int_0^{\pi} (2\cos x - 1)\,dx = -\pi$ and say what this number measures. [5]

Or *(a) Find where the curve $y = 1 - 2\sin x$ meets the x-axis for $0 \le x \le \pi$. (b) Find the area of the region bounded by the curve, the x-axis and the ordinates $x = 0$ and $x = \pi$. (c) Show that $\int_0^{\pi} (1 - 2\sin x)\,dx = \pi - 4$ and say what this number measures.*

(a) $2\cos x - 1 = 0$ gives $\cos x = \frac{1}{2}$, $x = \frac{\pi}{3}$; the curve is above the x-axis on $(0, \frac{\pi}{3})$ and below on $(\frac{\pi}{3}, \pi)$. (b) An anti derivative is $2\sin x - x$. $\int_0^{\pi/3} = \sqrt{3} - \frac{\pi}{3}$; $\int_{\pi/3}^{\pi} = (0 - \pi) - (\sqrt{3} - \frac{\pi}{3}) = -\sqrt{3} - \frac{2\pi}{3}$, numerical value $\sqrt{3} + \frac{2\pi}{3}$. Area $= 2\sqrt{3} + \frac{\pi}{3}$ square units. (c) $\int_0^{\pi} = (\sqrt{3} - \frac{\pi}{3}) + (-\sqrt{3} - \frac{2\pi}{3}) = -\pi$: the area above the axis less the area below it, not the area of the region.

- (a) $x = \frac{\pi}{3}$ with the sign on each side (1)
- anti derivative $2\sin x - x$ (1)
- pieces $\sqrt{3} - \frac{\pi}{3}$ and $-\sqrt{3} - \frac{2\pi}{3}$ (1)
- (b) area $2\sqrt{3} + \frac{\pi}{3}$ square units (1)
- (c) $-\pi$ as area above less area below (1)

Or:

(a) $\sin x = \frac{1}{2}$ gives $x = \frac{\pi}{6}$ and $\frac{5\pi}{6}$; the curve is above the x-axis on $(0, \frac{\pi}{6})$ and $(\frac{5\pi}{6}, \pi)$, below on $(\frac{\pi}{6}, \frac{5\pi}{6})$. (b) An anti derivative is $x + 2\cos x$. $\int_0^{\pi/6} = \frac{\pi}{6} + \sqrt{3} - 2$; $\int_{\pi/6}^{5\pi/6} = \frac{2\pi}{3} - 2\sqrt{3}$, numerical value $2\sqrt{3} - \frac{2\pi}{3}$; $\int_{5\pi/6}^{\pi} = \frac{\pi}{6} + \sqrt{3} - 2$. Area $= 4\sqrt{3} - 4 - \frac{\pi}{3}$ square units. (c) Adding the three signed pieces gives $\frac{\pi}{3} + 2\sqrt{3} - 4 + \frac{2\pi}{3} - 2\sqrt{3} = \pi - 4$: the area above the axis less the area below it.

- (a) $x = \frac{\pi}{6}, \frac{5\pi}{6}$ with the signs (1)
- anti derivative $x + 2\cos x$ (1)
- the three pieces (1)
- (b) area $4\sqrt{3} - 4 - \frac{\pi}{3}$ square units (1)
- (c) $\pi - 4$ as area above less area below (1)

33. (a) Using $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$, show that $\int_0^\pi x\,g(\sin x)\,dx = \frac{\pi}{2}\int_0^\pi g(\sin x)\,dx$ for a continuous function g. (b) Hence evaluate $\int_0^\pi x \sin x \cos^2 x\,dx$. [5]

Or *Evaluate* $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x}\,dx$.

(a) Let $I = \int_0^\pi x\,g(\sin x)\,dx$. By P_4, $I = \int_0^\pi (\pi - x)\,g(\sin(\pi - x))\,dx = \int_0^\pi (\pi - x)\,g(\sin x)\,dx = \pi\int_0^\pi g(\sin x)\,dx - I$. So $2I = \pi\int_0^\pi g(\sin x)\,dx$. (b) $\sin x \cos^2 x = \sin x(1 - \sin^2 x)$ is a function of $\sin x$, so the integral is $\frac{\pi}{2}\int_0^\pi \sin x \cos^2 x\,dx$. With $\cos x = t$, $\int_0^\pi \sin x \cos^2 x\,dx = \int_{-1}^1 t^2\,dt = \frac{2}{3}$. The value is $\frac{\pi}{2}\cdot\frac{2}{3} = \frac{\pi}{3}$.

- (a) applies P_4 and uses $\sin(\pi - x) = \sin x$ (1)
- (a) adds to reach $2I = \pi\int g(\sin x)\,dx$ (1)
- (b) recognises $\cos^2 x$ as a function of $\sin x$ and applies (a) (1)
- (b) $\int_0^\pi \sin x \cos^2 x\,dx = \frac{2}{3}$ by $\cos x = t$ (1)
- (b) value $\frac{\pi}{3}$ (1)

Or:

Let I be the integral. By P_4, $I = \int_0^{\pi/2} \frac{\cos^2 x}{\cos x + \sin x}\,dx$. Adding, $2I = \int_0^{\pi/2} \frac{dx}{\sin x + \cos x}$. Now $\sin x + \cos x = \sqrt{2}\sin(x + \frac{\pi}{4})$, so $2I = \frac{1}{\sqrt{2}}\int_0^{\pi/2} \operatorname{cosec}(x + \frac{\pi}{4})\,dx = \frac{1}{\sqrt{2}}[\log|\operatorname{cosec}(x + \frac{\pi}{4}) - \cot(x + \frac{\pi}{4})|]_0^{\pi/2}$. At $\frac{\pi}{2}$: $\operatorname{cosec}\frac{3\pi}{4} - \cot\frac{3\pi}{4} = \sqrt{2} + 1$; at 0: $\sqrt{2} - 1$. So $2I = \frac{1}{\sqrt{2}}\log\frac{\sqrt{2}+1}{\sqrt{2}-1} = \frac{2}{\sqrt{2}}\log(\sqrt{2}+1)$, and $I = \frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$.

- applies P_4 to get the $\cos^2 x$ form (1)
- adds to reach $2I = \int \frac{dx}{\sin x + \cos x}$ (1)
- writes $\sin x + \cos x = \sqrt{2}\sin(x + \frac{\pi}{4})$ (1)
- integrates cosec and substitutes the limits (1)
- value $\frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$ (1)

34. For $f(x) = x^4 - 4x^3 - 8x^2 + 48x + 1$: (a) find the intervals in which f is increasing and in which it is decreasing; (b) find the points of local maxima and local minima and the local maximum and minimum values. [5]

(a) $f'(x) = 4x^3 - 12x^2 - 16x + 48 = 4(x^2 - 4)(x - 3) = 4(x + 2)(x - 2)(x - 3)$, zero at $-2, 2, 3$. Signs: $(-\infty, -2)$ negative; $(-2, 2)$ positive; $(2, 3)$ negative; $(3, \infty)$ positive. Decreasing in $(-\infty, -2)$ and $(2, 3)$; increasing in $(-2, 2)$ and $(3, \infty)$. (b) $f''(x) = 12x^2 - 24x - 16$: $f''(-2) = 80 > 0$, $f''(2) = -16 < 0$, $f''(3) = 20 > 0$. Local minima at $x = -2$, value $f(-2) = 16 + 32 - 32 - 96 + 1 = -79$; local maxima at $x = 2$, value $f(2) = 16 - 32 - 32 + 96 + 1 = 49$; local minima at $x = 3$, value $f(3) = 81 - 108 - 72 + 144 + 1 = 46$.

- $f'(x) = 4(x + 2)(x - 2)(x - 3)$ (1)
- sign of f' and the four intervals (1)
- classifies $-2, 2, 3$ (by f'' or the sign change) (1)
- local minimum values -79 and 46 (1)
- local maximum value 49 (1)

35. A cube has one vertex at the origin O and the edges OA, OC and OD along the positive x, y and z-axes, each of length 2 units. F is the vertex opposite O. [5]

(a) Find the angle between the diagonal OF and the face diagonal AC.

(b) Find the shortest distance between OF and the edge through A parallel to OD.

(c) Find the shortest distance between OF and AC.

A(2, 0, 0), C(0, 2, 0), D(0, 0, 2), F(2, 2, 2). (a) OF has direction ratios 1, 1, 1 and AC has −2, 2, 0: −2 + 2 + 0 = 0, so the angle is 90°. (b) OF: $\vec{r} = \lambda(\hat{i} + \hat{j} + \hat{k})$; edge: $\vec{r} = 2\hat{i} + \mu\hat{k}$. $\vec{b_1} \times \vec{b_2} = \hat{i} - \hat{j}$, of magnitude $\sqrt{2}$; $(\vec{a_2} - \vec{a_1}) = 2\hat{i}$, dot product 2. $d = \frac{2}{\sqrt{2}} = \sqrt{2}$ units. (c) AC: $\vec{r} = 2\hat{i} + \mu(-2\hat{i} + 2\hat{j})$. $\vec{b_1} \times \vec{b_2} = |\hat{i}\ \hat{j}\ \hat{k};\ 1\ 1\ 1;\ -2\ 2\ 0| = -2\hat{i} - 2\hat{j} + 4\hat{k}$, of magnitude $2\sqrt{6}$; dot product with $2\hat{i}$ is −4. $d = \frac{4}{2\sqrt{6}} = \frac{2}{\sqrt{6}} = \frac{\sqrt{6}}{3}$ units.

- angle 90° from $a_1a_2 + b_1b_2 + c_1c_2 = 0$ (1)
- (b) $\vec{b_1} \times \vec{b_2} = \hat{i} - \hat{j}$ and $\vec{a_2} - \vec{a_1} = 2\hat{i}$ (1)
- (b) $d = \sqrt{2}$ (1)
- (c) $\vec{b_1} \times \vec{b_2} = -2\hat{i} - 2\hat{j} + 4\hat{k}$ (1)
- (c) $d = \frac{2}{\sqrt{6}}$ (1)

Section E (3 × 4 marks)

36. Read the passage and answer the questions that follow. [4]

A dairy cooperative near Anand sells milk, curd and ghee at two outlets. Daily sales are recorded in the matrix S, rows for Outlet 1 and Outlet 2, columns for milk (litres), curd (kg) and ghee (kg). The selling prices per unit form the column matrix P and the cost prices per unit the column matrix C, in rupees, in the same order.

Matrix	Entries
S	[[120, 40, 10], [90, 60, 15]]
P	[[56], [80], [600]]
C	[[48], [65], [520]]

(a) The order of the revenue matrix SP is [1]

(a) order 2 × 1 (b) order 1 × 2
(c) order 3 × 1 (d) order 2 × 3

Answer: (a) S is 2 × 3 and P is 3 × 1; the columns of S equal the rows of P, and SP is 2 × 1.

(b) The daily revenue of Outlet 2 is [1]

(a) ₹18,840 (b) ₹15,920
(c) ₹34,760 (d) ₹2,820

Answer: (a) Row 2 of S times P: 90(56) + 60(80) + 15(600) = 5040 + 4800 + 9000 = ₹18,840.

(c) Find the profit matrix S(P − C) and the total daily profit of the cooperative from both outlets. [2]

Or *Find the daily cost matrix SC, and use it with SP to find the profit of each outlet. Which law of matrix multiplication shows this equals S(P − C)?*

$P - C = \begin{pmatrix} 8 \\ 15 \\ 80 \end{pmatrix}$. $S(P - C) = \begin{pmatrix} 120(8) + 40(15) + 10(80) \\ 90(8) + 60(15) + 15(80) \end{pmatrix} = \begin{pmatrix} 960 + 600 + 800 \\ 720 + 900 + 1200 \end{pmatrix} = \begin{pmatrix} 2360 \\ 2820 \end{pmatrix}$. Total daily profit = 2360 + 2820 = ₹5,180.

- $P - C = \begin{pmatrix} 8 \\ 15 \\ 80 \end{pmatrix}$ and $S(P - C) = \begin{pmatrix} 2360 \\ 2820 \end{pmatrix}$ (1)
- total ₹5,180 (1)

Or:

$SC = \begin{pmatrix} 120(48) + 40(65) + 10(520) \\ 90(48) + 60(65) + 15(520) \end{pmatrix} = \begin{pmatrix} 5760 + 2600 + 5200 \\ 4320 + 3900 + 7800 \end{pmatrix} = \begin{pmatrix} 13560 \\ 16020 \end{pmatrix}$. $SP = \begin{pmatrix} 15920 \\ 18840 \end{pmatrix}$, so $SP - SC = \begin{pmatrix} 2360 \\ 2820 \end{pmatrix}$: ₹2,360 and ₹2,820. By the distributive law, SP − SC = S(P − C).

- $SC = \begin{pmatrix} 13560 \\ 16020 \end{pmatrix}$ (1)
- profits ₹2,360 and ₹2,820, distributive law named (1)

37. Read the passage and answer the questions that follow. [4]

A housing society in Jaipur fills its overhead tank with a pump whose output varies as the motor warms up. The caretaker notes that t minutes after switching on, water enters at the rate $r(t) = 3t^2 - 12t + 15$ litres per minute, for $0 \le t \le 6$. The volume added in the first x minutes is the area function $V(x) = \int_0^x r(t)\,dt$, and the volume added between two times is the definite integral of r over that interval.

(a) $V'(2)$ equals [1]

(a) 3 (b) 14
(c) 0 (d) 15

Answer: (a) By the first fundamental theorem $V'(x) = r(x)$, so $V'(2) = 12 - 24 + 15 = 3$ litres per minute. Option (b) is $V(2) = 8 - 24 + 30 = 14$, the volume rather than the rate; (c) is $r'(2)$; (d) is $r(0)$.

(b) The volume added in the first 3 minutes, in litres, is [1]

(a) 18 (b) 27
(c) 12 (d) 45

Answer: (a) $V(x) = x^3 - 6x^2 + 15x$, so $V(3) = 27 - 54 + 45 = 18$ litres.

(c) Find the volume of water added from t = 3 to t = 5. [2]

Or *Find the average rate of inflow over the first 6 minutes, $\frac{1}{6}\int_0^6 r(t)\,dt$.*

$\int_3^5 (3t^2 - 12t + 15)\,dt = [t^3 - 6t^2 + 15t]_3^5 = (125 - 150 + 75) - (27 - 54 + 45) = 50 - 18 = 32$ litres.

- anti derivative $t^3 - 6t^2 + 15t$ (1)
- 32 litres (1)

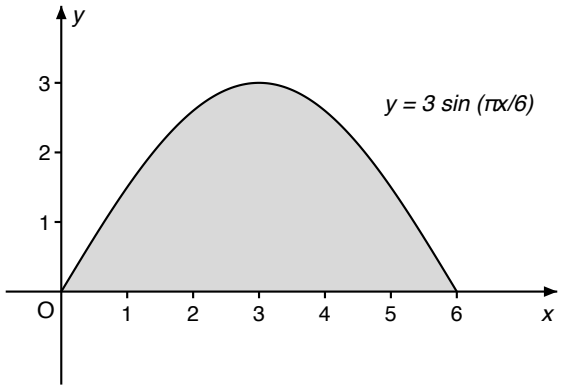
Or:

$\int_0^6 r(t)\,dt = 216 - 216 + 90 = 90$ litres, so the average rate is $\frac{90}{6} = 15$ litres per minute.

- $\int_0^6 r(t)\,dt = 90$ (1)
- average 15 litres per minute (1)

38. Read the passage and answer the questions that follow. [4]

A carpenter in Jodhpur is making a wooden gate for an arched opening in a haveli wall. Taking the left foot of the opening as the origin and measuring in metres, the floor is the x-axis and the top edge of the opening follows $y = 3\sin\frac{\pi x}{6}$ for $0 \le x \le 6$, as in the figure. The opening is the region bounded by this curve and the x-axis. To estimate the timber, he treats the opening as a large number of thin vertical strips and adds their areas.



(a) The width of the opening at a height of 1.5 m, in metres, is [1]

(a) 6 (b) 3

(c) 4 (d) 2

Answer: (c) $3\sin\frac{\pi x}{6} = 1.5$ gives $\sin\frac{\pi x}{6} = \frac{1}{2}$, so $\frac{\pi x}{6} = \frac{\pi}{6}$ or $\frac{5\pi}{6}$, that is $x = 1$ or $x = 5$. The width at that height is $5 - 1 = 4$ m. 6 is the width at the floor; 3 is the greatest height.

(b) The elementary area of a vertical strip of width dx at a distance x metres from the origin is [1]

(a) x dy (b) $3\sin\frac{\pi x}{6}\,dx$

(c) $\frac{\pi x}{6}\,dx$ (d) 3x dx

Answer: (b) A vertical strip has height $y = 3\sin\frac{\pi x}{6}$ and width dx, so $dA = y\,dx = 3\sin\frac{\pi x}{6}\,dx$. x dy is a horizontal strip.

(c) Find the area of the opening. [2]

$A = \int_0^6 3\sin\frac{\pi x}{6}\,dx = 3\cdot\frac{6}{\pi}\left[-\cos\frac{\pi x}{6}\right]_0^6 = \frac{18}{\pi}(1+1) = \frac{36}{\pi} \approx 11.46\ \text{m}^2$.

- sets up $\int_0^6 3\sin\frac{\pi x}{6}\,dx$ and integrates to $-\frac{18}{\pi}\cos\frac{\pi x}{6}$ (1)
- area $\frac{36}{\pi}\ \text{m}^2$ (about $11.46\ \text{m}^2$) (1)