CBSE Class 12 Mathematics: Sample Paper 2: Marking Scheme and Solutions

Based on NCERT · the board's 2026-27 pattern | Time: 3 hours · **Maximum marks: 80**

Section A (20 × 1 mark)

1. In a town in Assam, the probability that it rains on a given morning is 0.3, and the probability that the school bus is late on a rainy morning is 0.6. The probability that on a given morning it rains and the bus is late is [1]

(a) 0.9 (b) 0.18
(c) 0.5 (d) 0.42

Answer: (b) With R the event 'it rains' and L 'the bus is late', $P(R \cap L) = P(R)\,P(L|R) = 0.3 \times 0.6 = 0.18$. Option (a) adds; (c) divides; (d) uses $P(R') = 0.7$.

2. If $f(x) < 0$ for every x in [a, b], then the area of the region bounded by y = f(x), the x-axis and the ordinates x = a and x = b is [1]

(a) $\int_a^b f(x)\,dx$ (b) $-\int_a^b f(x)\,dx$
(c) $\int_a^b f(-x)\,dx$ (d) $-\int_b^a f(x)\,dx$

Answer: (b) Below the x-axis every strip has $y < 0$, so $\int_a^b f(x)\,dx$ is negative and the area is its numerical value, $\left|\int_a^b f(x)\,dx\right| = -\int_a^b f(x)\,dx$. $-\int_b^a f(x)\,dx$ equals $\int_a^b f(x)\,dx$, which is negative.

3. If $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + 4\hat{j} - \hat{k}$, then $\vec{a} \cdot \vec{b}$ is [1]

(a) 13 (b) 3
(c) −1 (d) −3

Answer: (d) $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3 = 6 - 8 - 1 = -3$. Taking the $\hat{k}$ term as +1 gives −1; taking (−2)(4) as +8 gives 13.

4. A spinner has 8 equal sectors numbered 1 to 8 and is spun once. E is the event 'the number is even', and G is an event made up of four of the eight numbers. E and G are independent exactly when the number of even numbers in G is [1]

(a) 2 (b) 4
(c) 1 (d) 3

Answer: (a) P(E) = 1/2 and P(G) = 4/8 = 1/2. If G contains m even numbers, $P(E \cap G) = m/8$. Independence needs $m/8 = 1/2 \times 1/2 = 1/4$, so m = 2; for example G = {1, 2, 3, 4}. Option (b) would make G = E, with $P(E \cap G) = 1/2$.

5. The general solution of $\frac{dy}{dx} = \frac{2xy}{1 + x^2}$, $y > 0$, is [1]

(a) $y = 1 + x^2 + C$ (b) $y = C(1 + x^2)^2$
(c) $y = C(1 + x^2)$ (d) $y = C \log (1 + x^2)$

Answer: (c) Separating, $dy/y = 2x\,dx/(1 + x^2)$. Integrating: $\log y = \log (1 + x^2) + \log C$, so $y = C(1 + x^2)$. Option (a) forgets that the constant multiplies after taking exponentials.

6. In A = {1, 2, 3, ..., 20}, R = {(a, b) : a − b is divisible by 6} is an equivalence relation. The number of equivalence classes of R that contain exactly 4 elements is [1]

(a) 4 (b) 6

(c) 2 (d) 3

Answer: (c) The classes are the six remainder classes on division by 6. Since $20 = 6 \times 3 + 2$, the remainders 1 and 2 occur four times ({1, 7, 13, 19}, {2, 8, 14, 20}) and the remainders 3, 4, 5, 0 three times. So 2 classes have 4 elements. 6 is the number of classes; 3 is the size of the others.

7. Which of the following points lies on the line $\frac{x-3}{2} = \frac{y+1}{-5} = \frac{z-4}{6}$? [1]

(a) (5, −4, 10) (b) (3, 1, 4)

(c) (2, −5, 6) (d) (5, −6, 10)

Answer: (d) A general point of the line is $(3 + 2\lambda, -1 - 5\lambda, 4 + 6\lambda)$; $\lambda = 1$ gives (5, −6, 10). For (b) the three ratios are 0, $-\frac{2}{5}$, 0; (c) reads the direction ratios as a point; for (a) the ratios are 1, $\frac{3}{5}$, 1.

8. The feasible region of a system of linear inequalities is said to be bounded if [1]

(a) it can be enclosed within a circle (b) it contains the origin

(c) it has at most four corner points (d) it lies in the first quadrant

Answer: (a) By definition a feasible region is bounded if it can be enclosed within a circle; otherwise it is unbounded. A region in the first quadrant, or one containing the origin, can still extend indefinitely, and a region with only two corner points can be unbounded.

9. For a real function f and a point c in its domain, which statement is always true? [1]

(a) If f is continuous at c, then f is differentiable at c (b) If f is differentiable at c, then f is continuous at c

(c) If f is not differentiable at c, then f is not continuous at c (d) If f is continuous at c, then $f'(c) = 0$

Answer: (b) Theorem 3: differentiability at c implies continuity at c. The converse fails: |x| is continuous at 0 but not differentiable there, which rules out (a) and (c).

10. The function $f(x) = 5 - 2x^3$ on R is [1]

(a) strictly increasing (b) strictly decreasing

(c) constant (d) neither increasing nor decreasing

Answer: (b) If $x_1 < x_2$, then $x_1^3 < x_2^3$, so $-2x_1^3 > -2x_2^3$ and $f(x_1) > f(x_2)$: f is strictly decreasing on R. $f'(x) = -6x^2$ is 0 at $x = 0$ only, which does not spoil strictness.

11. A is a square matrix of order 3 with $|A| = 4$. Then $|2A|$ is [1]

(a) 16 (b) 64

(c) 8 (d) 32

Answer: (d) For order n, $|kA| = k^n|A|$. Here n = 3, so $|2A| = 2^3 \times 4 = 32$. Writing $2|A|$ gives 8; $2^2|A|$ gives 16.

12. $\int \frac{dx}{x^2 + 25}$ equals [1]

(a) $\frac{1}{25}\tan^{-1}\frac{x}{5} + C$ (b) $\frac{1}{5}\tan^{-1}\frac{x}{5} + C$

(c) $\tan^{-1}\frac{x}{5} + C$ (d) $\frac{1}{10}\log\left|\frac{x-5}{x+5}\right| + C$

Answer: (b) $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$ with a = 5. Option (d) is the form for $x^2 - 25$.

13. A bounded feasible region has corner points (2, 1), (5, 1), (5, 4) and (1, 3). The minimum value of Z = 4x + 3y on the region is [1]

(a) 11 (b) 13

(c) 23 (d) 32

Answer: (a) Z at the corners: (2, 1) → 11, (5, 1) → 23, (5, 4) → 32, (1, 3) → 13. For a bounded region the smallest corner value is the minimum: 11 at (2, 1). 13 is the value at (1, 3), the corner with the smallest x, and 32 is the maximum.

14. The vectors $2\hat{i} + \lambda\hat{j} - 3\hat{k}$ and $4\hat{i} - \hat{j} + 2\hat{k}$ are perpendicular when λ equals [1]

(a) 2 (b) −2

(c) 14 (d) −14

Answer: (a) Perpendicular means the scalar product is 0: 8 − λ − 6 = 0, so λ = 2. Adding −6 as +6 gives 14.

15. In the set A = {1, 2, 3, 4, 5}, which of the following is the empty relation? [1]

(a) R = {(a, b) : a + b ≥ 2} (b) R = {(a, b) : a + b = 11}

(c) R = {(a, b) : a − b = 4} (d) R = {(a, b) : ab = 5}

Answer: (b) The largest sum of two elements of A is 5 + 5 = 10, so no pair has a + b = 11 and R = ϕ. (a) holds for every pair, so it is the universal relation; (c) contains (5, 1); (d) contains (1, 5) and (5, 1).

16. If $\cos^{-1} x = 5\pi/6$, then $\sin^{-1} x$ equals [1]

(a) π/3 (b) −π/3

(c) −π/6 (d) 2π/3

Answer: (b) $x = \cos(5\pi/6) = -\sqrt{3}/2$. $\sin^{-1}(-\sqrt{3}/2)$ must lie in $[-\pi/2, \pi/2]$, and $\sin(-\pi/3) = -\sqrt{3}/2$, so $\sin^{-1} x = -\pi/3$.

17. A parcel from Surat to Shillong passes through three hubs. The probability that it clears the first hub on time is 0.9; given that it has cleared the first on time, the probability that it clears the second on time is 0.8; given that it has cleared both on time, the probability that it clears the third on time is 0.75. The probability that it clears all three hubs on time is [1]

(a) 0.75 (b) 0.72

(c) 0.005 (d) 0.54

Answer: (d) By the multiplication rule for three events, $P(E \cap F \cap G) = P(E)\,P(F|E)\,P(G|EF) = 0.9 \times 0.8 \times 0.75 = 0.54$. Option (b) stops after two hubs; (c) multiplies the three complements.

18. A line through the origin is perpendicular to the z-axis, makes an angle of 30° with the positive direction of the x-axis and an acute angle with the positive direction of the y-axis. Its direction cosines are [1]

(a) $\frac{\sqrt{3}}{2}, 0, \frac{1}{2}$ (b) $\frac{1}{2}, \frac{\sqrt{3}}{2}, 0$

(c) $\frac{\sqrt{3}}{2}, \frac{1}{2}, 0$ (d) $\frac{\sqrt{3}}{2}, \frac{1}{2}, 1$

Answer: (c) Perpendicular to the z-axis: γ = 90°, n = 0. $l = \cos 30° = \frac{\sqrt{3}}{2}$. Then $m^2 = 1 - \frac{3}{4} - 0 = \frac{1}{4}$ and $m = \frac{1}{2}$ (acute angle). Option (b) swaps l and m; (d) puts n = cos 0°.

19. Assertion (A): $f(x) = x^3 - 4x + 7$ is continuous at $x = 2$. Reason (R): Every polynomial function is continuous. [1]

(a) Both A and R are true, and R is the correct explanation of A.
(b) Both A and R are true, but R is not the correct explanation of A.
(c) A is true, but R is false.
(d) A is false, but R is true.

Answer: (a) f is a polynomial, and a polynomial is continuous at every real number: $\lim_{x\to 2} f(x) = 8 - 8 + 7 = 7 = f(2)$. R is true and is the reason A holds.

20. Assertion (A): The differential equation $\frac{dy}{dx} = \frac{y - x}{y + x}$ is homogeneous. Reason (R): The function $x^2 - 3xy + y^2$ is a homogeneous function of degree 2. [1]

(a) Both A and R are true, and R is the correct explanation of A.
(b) Both A and R are true, but R is not the correct explanation of A.
(c) A is true, but R is false.
(d) A is false, but R is true.

Answer: (b) A: $F(\lambda x, \lambda y) = \lambda(y - x)/(\lambda(y + x)) = \lambda^0 F(x, y)$, so F has degree zero and the equation is homogeneous. R: each term has degree 2, so R is true. But A rests on F being of degree zero, a fact about a different function; R does not explain A.

Section B (5 × 2 marks)

21. A nursery in Pune sells seed packets of two varieties: 70% are of variety X and 30% of variety Y. A seed of variety X germinates with probability 0.8 and one of variety Y with probability 0.6. A seed is taken at random from the nursery's stock. Find the probability that it does not germinate. [2]

Or *In a school library in Mysuru, 40% of the books are fiction. In a month, a fiction book is borrowed with probability 0.3 and any other book with probability 0.1. A book is picked at random from the shelves. Find the probability that it was borrowed in the month.*

X and Y form a partition. By the theorem of total probability, $P(G) = 0.7 \times 0.8 + 0.3 \times 0.6 = 0.56 + 0.18 = 0.74$, so $P(\text{does not germinate}) = 1 - 0.74 = 0.26$. (Equally, $0.7 \times 0.2 + 0.3 \times 0.4 = 0.26$.)

- $P(G) = P(X)\,P(G|X) + P(Y)\,P(G|Y)$ set up (1)
- 0.74, so the answer is 0.26 (1)

Or:

Fiction (F) and non-fiction (N) form a partition. $P(B) = P(F)\,P(B|F) + P(N)\,P(B|N) = 0.4 \times 0.3 + 0.6 \times 0.1 = 0.12 + 0.06 = 0.18$.

- total probability set up with $P(N) = 0.6$ (1)
- 0.18 (1)

22. Find the value of $\tan^{-1}(\tan 2) + \cos^{-1}(\cos 5)$, the angles being in radians. [2]

$\pi/2 < 2 < \pi$, and $\tan 2 = \tan(2 - \pi)$ with $2 - \pi \in (-\pi/2, \pi/2)$, so $\tan^{-1}(\tan 2) = 2 - \pi$. $3\pi/2 < 5 < 2\pi$, and $\cos 5 = \cos(2\pi - 5)$ with $2\pi - 5 \in [0, \pi]$, so $\cos^{-1}(\cos 5) = 2\pi - 5$. Sum $= (2 - \pi) + (2\pi - 5) = \pi - 3$.

- $\tan^{-1}(\tan 2) = 2 - \pi$, with the branch stated (1)
- $\cos^{-1}(\cos 5) = 2\pi - 5$, and sum $\pi - 3$ (1)

23. If $|\vec{a}| = 5$, $|\vec{b}| = 3$ and $|\vec{a} - \vec{b}| = 7$, find $|\vec{a} + \vec{b}|$. [2]

$|\vec{a} - \vec{b}|^2 = 25 + 9 - 2\vec{a} \cdot \vec{b} = 49$ gives $\vec{a} \cdot \vec{b} = -\frac{15}{2}$. Then $|\vec{a} + \vec{b}|^2 = 25 + 9 + 2(-\frac{15}{2}) = 19$, so $|\vec{a} + \vec{b}| = \sqrt{19}$.

- $\vec{a} \cdot \vec{b} = -\frac{15}{2}$ (1)
- $|\vec{a} + \vec{b}| = \sqrt{19}$ (1)

24. If $x = e^t + e^{-t}$ and $y = e^t - e^{-t}$, show that $\frac{dy}{dx} = \frac{x}{y}$. [2]

$dx/dt = e^t - e^{-t} = y$ and $dy/dt = e^t + e^{-t} = x$, so $dy/dx = (dy/dt)/(dx/dt) = x/y$ (for $t \neq 0$, where $y \neq 0$).

- $dx/dt = y$ and $dy/dt = x$ (1)
- $dy/dx = x/y$ (1)

25. Find x, y and z if the matrix $\begin{pmatrix} 0 & 2x-1 & 4 \\ 3 & 0 & y+2 \\ z & -5 & 0 \end{pmatrix}$ is skew symmetric. [2]

Or *Find a, b and c if the matrix* $\begin{pmatrix} 0 & a+1 & -2 \\ 4 & b & c-3 \\ 2 & 6 & 0 \end{pmatrix}$ *is skew symmetric.*

For a skew symmetric matrix $a_{ji} = -a_{ij}$. From $a_{21} = 3$: $a_{12} = 2x - 1 = -3$, so $x = -1$. From $a_{32} = -5$: $a_{23} = y + 2 = 5$, so $y = 3$. From $a_{13} = 4$: $a_{31} = z = -4$. The diagonal elements are already 0.

- uses $a_{ji} = -a_{ij}$ (1)
- $x = -1$, $y = 3$, $z = -4$ (1)

Or:

For a skew symmetric matrix $a_{ji} = -a_{ij}$, and the diagonal is zero. $a + 1 = -4$, so $a = -5$. b is a diagonal element, so $b = 0$. $c - 3 = -6$, so $c = -3$. The pair $a_{13} = -2$, $a_{31} = 2$ already agrees.

- uses $a_{ji} = -a_{ij}$ and zero diagonal (1)
- $a = -5$, $b = 0$, $c = -3$ (1)

Section C (6 × 3 marks)

26. Kabir has at most 3 hours a day for exercise. He walks for x hours and cycles for y hours, and on his doctor's advice cycles for no longer than he walks. Walking burns 300 kcal an hour and cycling 500 kcal an hour. Formulate a linear programming problem to maximise the energy he burns, and solve it graphically. [3]

Maximise $Z = 300x + 500y$ subject to $x + y \leq 3$, $y \leq x$, $x \geq 0$, $y \geq 0$. The line $y = x$ passes through the origin; testing (1, 0) shows $y \leq x$ is the side below it. The bounded region has corners (0, 0), (3, 0) and (3/2, 3/2), where $y = x$ meets $x + y = 3$. Z there: 0, 900, 1200. The maximum is 1200 kcal, by walking 1.5 hours and cycling 1.5 hours.

- formulation: $Z = 300x + 500y$; $x + y \leq 3$, $y \leq x$, $x, y \geq 0$ (1)
- corners (0, 0), (3, 0), (3/2, 3/2) (1)
- maximum 1200 kcal at 1.5 h walking, 1.5 h cycling (1)

27. Show that $f : R \to R$, $f(x) = 2x + |x|$, is one-one and onto. [3]

Or *Show that $f : R \to R$, $f(x) = x + |x|$, is neither one-one nor onto. Find a subset D of R, containing 0, such that f, with domain D and co-domain $[0, \infty)$, is one-one and onto.*

$f(x) = 3x$ for $x \geq 0$ and $f(x) = x$ for $x < 0$; $f(x) \geq 0$ when $x \geq 0$ and $f(x) < 0$ when $x < 0$. One-one: if $f(x_1) = f(x_2)$, the common value has one sign, so x_1, x_2 are on the same side of 0; then $3x_1 = 3x_2$ or $x_1 = x_2$, giving $x_1 = x_2$. Onto: for $y \geq 0$, $x = y/3 \geq 0$ and $f(x) = y$; for $y < 0$, $x = y < 0$ and $f(x) = y$. So f is one-one and onto.

- f written piecewise, with the sign of f(x) noted (1)
- one-one by cases (1)
- onto: preimages y/3 and y (1)

Or:

$f(x) = 2x$ for $x \geq 0$ and $f(x) = 0$ for $x < 0$. $f(-1) = f(-2) = 0$, so f is not one-one. $f(x) \geq 0$ for every x, so -1 is not the image of any x: f is not onto. On $D = [0, \infty)$, $f(x) = 2x$: $2x_1 = 2x_2$ gives $x_1 = x_2$, and every $y \geq 0$ is $f(y/2)$ with $y/2 \in D$, so $f : [0, \infty) \to [0, \infty)$ is one-one and onto.

- not one-one: $f(-1) = f(-2)$ (1)
- not onto: negative numbers are not images (1)
- $D = [0, \infty)$, with one-one and onto shown (1)

28. Three NCC cadets fire one shot each at a target, independently. They hit with probabilities $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. Find the probability that exactly two of them hit the target. [3]

Or *A water pump works if its motor works and at least one of its two valves works. The motor works with probability 0.9 and the valves with probabilities 0.8 and 0.5, all independently. Find the probability that the pump works.*

Let A, B, C be the hits; their complements are independent too. $P(A\ B\ C') = 1/2 \times 1/3 \times 3/4 = 3/24$; $P(A\ B'\ C) = 1/2 \times 2/3 \times 1/4 = 2/24$; $P(A'\ B\ C) = 1/2 \times 1/3 \times 1/4 = 1/24$. These are mutually exclusive, so P(exactly two) = 6/24 = 1/4.

- the three mutually exclusive cases written (1)
- each probability by independence: 3/24, 2/24, 1/24 (1)
- sum 1/4 (1)

Or:

P(at least one valve works) = 1 − P(both fail) = $1 - 0.2 \times 0.5 = 0.9$, using the independence of the complements. The motor is independent of the valves, so P(pump works) = $0.9 \times 0.9 = 0.81$.

- P(both valves fail) = $0.2 \times 0.5 = 0.1$ (1)
- P(at least one valve works) = 0.9 (1)
- × 0.9 for the motor: 0.81 (1)

29. Show that the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{-1}$ and $\frac{x-2}{1} = \frac{y-4}{-2} = \frac{z+2}{3}$ intersect, and find their point of intersection. [3]

General points: $(1 + 2\lambda, -1 + 3\lambda, 2 - \lambda)$ and $(2 + \mu, 4 - 2\mu, -2 + 3\mu)$. Equating x and y: $2\lambda - \mu = 1$ and $3\lambda + 2\mu = 5$, so $\lambda = 1$, $\mu = 1$. Then z: $2 - 1 = 1$ and $-2 + 3 = 1$, equal, so the lines intersect. The point is (3, 2, 1).

- general points of both lines (1)
- $\lambda = 1$, $\mu = 1$ from two coordinates, third coordinate verified (1)
- point of intersection (3, 2, 1) (1)

30. A farmer has to fence a rectangular plot of area 800 m^2 that lies along a straight canal; the side along the canal needs no fence. Find the dimensions that use the least fencing, and that length. [3]

Or *An open tank with a square base is to hold 32 m^3. Find the dimensions for which the area of sheet metal needed (base and four sides) is least.*

Let each side perpendicular to the canal be w m; the side along the canal is 800/w m. Fencing $L(w) = 2w + 800/w$. $L'(w) = 2 - 800/w^2 = 0$ at $w = 20$ ($w > 0$). $L''(w) = 1600/w^3 > 0$, a minimum. Dimensions 20 m by 40 m (40 m along the canal); least fencing 40 + 40 = 80 m.

- $L(w) = 2w + 800/w$ (1)
- $L'(w) = 0$ at $w = 20$ with $L'' > 0$ (1)
- 20 m × 40 m, fencing 80 m (1)

Or:

Base side x m, depth $32/x^2$ m. Area $S(x) = x^2 + 4x(32/x^2) = x^2 + 128/x$. $S'(x) = 2x - 128/x^2 = 0$ gives $x^3 = 64$, $x = 4$. $S''(x) = 2 + 256/x^3 > 0$, a minimum. Base 4 m × 4 m, depth 2 m (sheet needed 48 m^2).

- $S(x) = x^2 + 128/x$ (1)
- $S'(x) = 0$ at $x = 4$ with $S'' > 0$ (1)
- base 4 m by 4 m, depth 2 m (1)

31. Show that the system $x + 2y - z = 3$, $2x - y + z = 1$, $3x + y = 6$ has no solution, by computing $|A|$ and (adj A)B. [3]

$A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & -1 & 1 \\ 3 & 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 3 \\ 1 \\ 6 \end{pmatrix}$. $|A| = 1(0 - 1) - 2(0 - 3) + (-1)(2 + 3) = -1 + 6 - 5 = 0$, so A is singular. Cofactors: $A_{11} = -1$, $A_{12} = 3$, $A_{13} = 5$, $A_{21} = -1$, $A_{22} = 3$, $A_{23} = 5$, $A_{31} = 1$, $A_{32} = -3$, $A_{33} = -5$, so adj $A = \begin{pmatrix} -1 & -1 & 1 \\ 3 & 3 & -3 \\ 5 & 5 & -5 \end{pmatrix}$. (adj A)B $= \begin{pmatrix} -3 - 1 + 6 \\ 9 + 3 - 18 \\ 15 + 5 - 30 \end{pmatrix} = \begin{pmatrix} 2 \\ -6 \\ -10 \end{pmatrix} \neq O$, so the system is inconsistent.

- $|A| = 0$ (1)
- adj A (1)
- (adj A)B ≠ O, so no solution (1)

Section D (4 × 5 marks)

32. (a) Sketch the curve $y = x^2 - 2x - 3$ for $-2 \le x \le 4$ and mark the points where it meets the x-axis. [5] (b) Find the area of the region bounded by the curve, the x-axis and the ordinates $x = -2$ and $x = 4$. (c) Evaluate $\int_{-2}^{4} (x^2 - 2x - 3)\,dx$ and explain why it differs from your answer to (b).

Or *(a) Sketch the curve $y = x^2 - x - 2$ for $-2 \le x \le 3$ and mark the points where it meets the x-axis. (b) Find the area of the region bounded by the curve, the x-axis and the ordinates $x = -2$ and $x = 3$. (c) Evaluate $\int_{-2}^{3} (x^2 - x - 2)\,dx$ and explain why it differs from your answer to (b).*

(a) $y = (x - 3)(x + 1)$ meets the x-axis at $x = -1$ and $x = 3$; it is an upward parabola with vertex $(1, -4)$, above the axis on $(-2, -1)$ and $(3, 4)$ and below it on $(-1, 3)$. (b) $F(x) = \frac{x^3}{3} - x^2 - 3x$: $F(-2) = -\frac{2}{3}$, $F(-1) = \frac{5}{3}$, $F(3) = -9$, $F(4) = -\frac{20}{3}$. The pieces are $\frac{7}{3}$, $-\frac{32}{3}$ and $\frac{7}{3}$, so the area is $\frac{7}{3} + \frac{32}{3} + \frac{7}{3} = \frac{46}{3}$ square units. (c) $\int_{-2}^{4} = F(4) - F(-2) = -6 = \frac{7}{3} - \frac{32}{3} + \frac{7}{3}$: the integral counts the part below the x-axis as negative, while an area takes its numerical value.

- (a) zeros -1 and 3 and a sketch showing the vertex $(1, -4)$ (1)
- anti derivative and the split at -1 and 3 (1)
- the three pieces $\frac{7}{3}$, $-\frac{32}{3}$, $\frac{7}{3}$ (1)
- (b) area $\frac{46}{3}$ square units (1)
- (c) -6, with the reason that the part below the axis is subtracted (1)

Or:

(a) $y = (x - 2)(x + 1)$ meets the x-axis at $x = -1$ and $x = 2$; vertex $(\frac{1}{2}, -\frac{9}{4})$; above the axis on $(-2, -1)$ and $(2, 3)$, below on $(-1, 2)$. (b) $F(x) = \frac{x^3}{3} - \frac{x^2}{2} - 2x$: $F(-2) = -\frac{2}{3}$, $F(-1) = \frac{7}{6}$, $F(2) = -\frac{10}{3}$, $F(3) = -\frac{3}{2}$. The pieces are $\frac{11}{6}$, $-\frac{9}{2}$ and $\frac{11}{6}$, so the area is $\frac{11}{6} + \frac{9}{2} + \frac{11}{6} = \frac{49}{6}$ square units. (c) $\int_{-2}^{3} = F(3) - F(-2) = -\frac{5}{6}$: the part below the x-axis enters the integral as negative, while an area takes its numerical value.

- (a) zeros -1 and 2 and a sketch showing the vertex $(\frac{1}{2}, -\frac{9}{4})$ (1)
- anti derivative and the split at -1 and 2 (1)
- the three pieces $\frac{11}{6}$, $-\frac{9}{2}$, $\frac{11}{6}$ (1)
- (b) area $\frac{49}{6}$ square units (1)
- (c) $-\frac{5}{6}$, with the reason (1)

33. (a) Find $\int \log (x^2 + 1)\, dx$. (b) Find $\int e^x (\cot x - \text{cosec}^2 x)\, dx$. [5]

Or *(a) Find $\int x^2 \cos x\, dx$. (b) Find $\int e^x (\text{cosec } x - \text{cosec } x \cot x)\, dx$.*

(a) By parts with 1 as the second function: $\int \log (x^2 + 1)\, dx = x \log (x^2 + 1) - \int \frac{2x^2}{x^2 + 1} dx = x \log (x^2 + 1) - \int (2 - \frac{2}{x^2 + 1}) dx = x \log (x^2 + 1) - 2x + 2 \tan^{-1} x + C$. (b) With $f(x) = \cot x$, $f'(x) = -\text{cosec}^2 x$, so the integrand is $e^x[f(x) + f'(x)]$ and the integral is $e^x \cot x + C$.

- (a) takes 1 as the second function and integrates by parts (1)
- (a) splits $\frac{2x^2}{x^2 + 1}$ as $2 - \frac{2}{x^2 + 1}$ (1)
- (a) $x \log (x^2 + 1) - 2x + 2 \tan^{-1} x + C$ (1)
- (b) identifies $f(x) = \cot x$ and $f'(x) = -\text{cosec}^2 x$ (1)
- (b) $e^x \cot x + C$ (1)

Or:

(a) With x^2 first: $\int x^2 \cos x\, dx = x^2 \sin x - \int 2x \sin x\, dx$, and $\int 2x \sin x\, dx = -2x \cos x + 2 \sin x$. So the integral is $x^2 \sin x + 2x \cos x - 2 \sin x + C$. (b) The integrand has the form $e^x[f(x) + f'(x)]$ with $f(x) = \text{cosec } x$, since $f'(x) = -\text{cosec } x \cot x$; so the integral is $e^x \text{cosec } x + C$.

- (a) first integration by parts with x^2 as first function (1)
- (a) second integration by parts on $2x \sin x$ (1)
- (a) $x^2 \sin x + 2x \cos x - 2 \sin x + C$ (1)
- (b) recognises $e^x[f(x) + f'(x)]$ with $f = \text{cosec } x$ (1)
- (b) $e^x \text{cosec } x + C$ (1)

34. Show that the differential equation $(x^2 + 3xy + y^2)\, dx - x^2\, dy = 0$ is homogeneous, and find its particular solution given that $y = 0$ when $x = 1$ (take $x > 0$, $x < e$). [5]

$dy/dx = (x^2 + 3xy + y^2)/x^2 = F(x, y)$. $F(\lambda x, \lambda y) = \lambda^2(x^2 + 3xy + y^2)/(\lambda^2 x^2) = \lambda^0 F(x, y)$: homogeneous. Put $y = vx$: $v + x\, dv/dx = 1 + 3v + v^2$, so $x\, dv/dx = v^2 + 2v + 1 = (v + 1)^2$. Separating, $dv/(v + 1)^2 = dx/x$; integrating, $-1/(v + 1) = \log x + C$. At $x = 1$, $y = 0$, so $v = 0$: $-1 = C$. Hence $1/(v + 1) = 1 - \log x$, $v + 1 = 1/(1 - \log x)$, and $v = \log x/(1 - \log x)$. So $y = x \log x/(1 - \log x)$.

- shows $F(\lambda x, \lambda y) = \lambda^0 F(x, y)$ (1)
- $y = vx$ gives $x\, dv/dx = (v + 1)^2$ (1)
- $-\frac{1}{v + 1} = \log x + C$ (1)
- $C = -1$ from the condition (1)
- $y = \frac{x \log x}{1 - \log x}$ (1)

35. The points A(1, 0, 2), B(3, 2, 3), C(4, 4, 1) and D(2, 2, 0) are given. (a) Show that ABCD is a rhombus. [5] (b) Show that its diagonals are perpendicular. (c) Find its area in two ways: from $\vec{AB} \times \vec{AD}$, and from the diagonals. (d) Find $\cos \angle DAB$.

(a) $\vec{AB} = 2\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{DC} = 2\hat{i} + 2\hat{j} + \hat{k}$, so $\vec{AB} = \vec{DC}$ and ABCD is a parallelogram; $|\vec{AB}| = 3$ and $\vec{AD} = \hat{i} + 2\hat{j} - 2\hat{k}$ has $|\vec{AD}| = 3$, so adjacent sides are equal: a rhombus. (b) $\vec{AC} = 3\hat{i} + 4\hat{j} - \hat{k}$, $\vec{BD} = -\hat{i} - 3\hat{k}$; $\vec{AC} \cdot \vec{BD} = -3 + 0 + 3 = 0$. (c) $\vec{AB} \times \vec{AD} = |\hat{i}\ \hat{j}\ \hat{k};\ 2\ 2\ 1;\ 1\ 2\ -2| = -6\hat{i} + 5\hat{j} + 2\hat{k}$, area $\sqrt{65}$. From the diagonals, $\frac{1}{2}|\vec{AC}||\vec{BD}| = \frac{1}{2}\sqrt{26}\sqrt{10} = \sqrt{65}$, the same. (d) $\cos \angle DAB = \frac{\vec{AB} \cdot \vec{AD}}{9} = \frac{2 + 4 - 2}{9} = \frac{4}{9}$.

- $\vec{AB} = \vec{DC}$ and $|\vec{AB}| = |\vec{AD}| = 3$ (1)
- $\vec{AC} \cdot \vec{BD} = 0$ (1)
- $\vec{AB} \times \vec{AD} = -6\hat{i} + 5\hat{j} + 2\hat{k}$, area $\sqrt{65}$ (1)
- $\frac{1}{2}|\vec{AC}||\vec{BD}| = \sqrt{65}$ (1)
- $\cos \angle DAB = \frac{4}{9}$ (1)

Section E (3 × 4 marks)

36. Read the passage and answer the questions that follow. [4]

A farmer near Nashik marks a triangular plot on a grid map, one unit standing for 10 m. The corner posts are at P(2, 1), Q(8, 3) and R(4, 7). A water tap is to be fixed at T(5, k) on the straight fence PQ, and a straight pipe is to run along the side QR. He uses determinants for every calculation.

(a) The area of the plot PQR, in square units, is [1]

(a) 16 (b) 32

(c) 20 (d) 8

Answer: (a) $\frac{1}{2}|2(3-7)+8(7-1)+4(1-3)| = \frac{1}{2}|-8+48-8| = 16$.

(b) The value of k for which T(5, k) lies on PQ is [1]

(a) $\frac{5}{2}$ (b) 3

(c) 2 (d) −2

Answer: (c) P, Q, T collinear: $2(3-k)+8(k-1)+5(1-3)=0$, so $6-2k+8k-8-10=0$, $6k=12$, $k=2$.

(c) Find the equation of the line QR using determinants. [2]

Or *A point S(k, 1) is chosen so that the area of triangle PQS is 9 square units. Find the possible values of k.*

For (x, y) on QR: $\begin{vmatrix} x & y & 1 \\ 8 & 3 & 1 \\ 4 & 7 & 1 \end{vmatrix} = 0$, so $x(3-7)-y(8-4)+(56-12)=0$, i.e. $-4x-4y+44=0$, or $x+y=11$. Q: $8+3=11$ and R: $4+7=11$.

- determinant with (x, y), Q, R set to zero (1)
- $x + y = 11$ (1)

Or:

$\frac{1}{2}|2(3-1)+8(1-1)+k(1-3)| = 9$, so $|4-2k| = 18$. Taking both signs, $4-2k=18$ or $4-2k=-18$, so $k=-7$ or $k=11$.

- area equation with both signs (1)
- $k = -7$ or 11 (1)

37. Read the passage and answer the questions that follow. [4]

A design student in Pune is drawing a curve for a logo on a coordinate grid. She wants the slope of the tangent at every point (x, y) of the curve to equal 2x − y, so the curve satisfies $\frac{dy}{dx} = 2x - y$. Rewritten as $\frac{dy}{dx} + y = 2x$, this is a first order linear differential equation, and every curve with this property belongs to one family of solution curves, one for each value of an arbitrary constant C. She first wants the member of the family that passes through the point (0, 1).

(a) The integrating factor of $\frac{dy}{dx} + y = 2x$ is [1]

(a) e^{x} (b) e^{-x}

(c) e^{2x} (d) x

Answer: (a) P = 1, so I.F. = $e^{\int 1\,dx} = e^{x}$.

(b) The general solution is $y = 2x - 2 + Ce^{-x}$. For the curve through (0, 1), the value of C is [1]

(a) 3 (b) 1

(c) −1 (d) 2

Answer: (a) At x = 0, y = 1: 1 = 0 − 2 + C, so C = 3. Option (b) forgets the constant term −2.

(c) Show how the general solution $y = 2x - 2 + Ce^{-x}$ is obtained from the integrating factor. [2]

Or *Find the member of the family that passes through (0, −2), and show that it is a straight line whose slope agrees with 2x − y at every point.*

Multiplying by e^{x}: $d/dx\,(y\,e^{x}) = 2x\,e^{x}$. By parts, $\int 2x\,e^{x}\,dx = 2x\,e^{x} - 2e^{x} + C$. So $y\,e^{x} = (2x - 2)\,e^{x} + C$ and $y = 2x - 2 + Ce^{-x}$.

- $y\,e^{x} = \int 2x\,e^{x}\,dx$ (1)
- integrates by parts to $y = 2x - 2 + Ce^{-x}$ (1)

Or:

At (0, −2): −2 = −2 + C, so C = 0 and the curve is y = 2x − 2, a straight line of slope 2. At any point on it, 2x − y = 2x − (2x − 2) = 2, equal to the slope.

- C = 0: y = 2x − 2 (1)
- shows 2x − y = 2 = slope at every point (1)

38. Read the passage and answer the questions that follow. [4]

In a physics laboratory in Kolkata, a small block is attached to a light spring on a smooth table. Students pull the block 3 cm from its rest position and push it, so that it starts with a speed of 8 cm per second. If x cm is the displacement of the block from rest after t seconds, its motion is described by the differential equation $\frac{d^2x}{dt^2} + 16x = 0$. The class wants to find the solution of this equation that fits the starting position and the starting speed.

(a) The order and the degree of the differential equation are [1]

(a) order 2, degree 1 (b) order 1, degree 2

(c) order 2, degree 2 (d) order 2, degree not defined

Answer: (a) The highest derivative is d^2x/dt^2, so the order is 2; the equation is a polynomial in it with power 1, so the degree is 1.

(b) Which of the following is the general solution of the equation (A, B arbitrary constants)? [1]

(a) $x = A \cos 4t + B \sin 4t$ (b) $x = A \cos 16t + B \sin 16t$

(c) $x = A\, e^{4t} + B\, e^{-4t}$ (d) $x = A \cos 4t$

Answer: (a) For $x = A \cos 4t + B \sin 4t$, $d^2x/dt^2 = -16(A \cos 4t + B \sin 4t) = -16x$, and it carries two arbitrary constants for a second order equation. (b) gives $-256x$; (c) gives $+16x$; (d) has only one constant.

(c) Find the particular solution with $x = 3$ and $\frac{dx}{dt} = 8$ at $t = 0$. [2]

$x = A \cos 4t + B \sin 4t$. At $t = 0$, $x = A = 3$. $dx/dt = -4A \sin 4t + 4B \cos 4t$; at $t = 0$ it is $4B = 8$, so $B = 2$. The particular solution is $x = 3 \cos 4t + 2 \sin 4t$.

- $A = 3$ from $x(0) = 3$ (1)
- $B = 2$ from $x'(0) = 8$; $x = 3 \cos 4t + 2 \sin 4t$ (1)