Class 11 Mathematics · Chapter 10: Conic Sections

Answer key and marking scheme · 20 marks · about 40 minutes · based on NCERT

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1. Which of the following parabolas has its focus on the negative y-axis? [1]

(a) $y^2 = -6x$ (b) $x^2 = 6y$

(c) $x^2 = -6y$ (d) $y^2 = 6x$

Answer: (c) $x^2 = -4ay$ has focus $(0, -a)$. Here $x^2 = -6y$ gives $a = \frac{3}{2}$ and focus $(0, -\frac{3}{2})$. The others have foci $(-\frac{3}{2}, 0)$, $(0, \frac{3}{2})$ and $(\frac{3}{2}, 0)$.

2. The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle that passes through the origin exactly when [1]

(a) $g = 0$ (b) $g = f$

(c) $c = 0$ (d) $g^2 + f^2 = c$

Answer: (c) $(0, 0)$ lies on the circle when $0 + 0 + 0 + 0 + c = 0$, i.e. $c = 0$ (as in $x^2 + y^2 - ax - by = 0$). $g = 0$ puts the centre on the y-axis, $g = f$ puts it on the line $y = x$, and $g^2 + f^2 = c$ makes the radius zero.

3. The point $(1, k)$ lies inside the circle $x^2 + y^2 = 10$ exactly when [1]

(a) $-3 < k < 3$ (b) $k < 3$

(c) $k = \pm 3$ (d) $-9 < k < 9$

Answer: (a) Inside means $1^2 + k^2 < 10$, i.e. $k^2 < 9$, so $-3 < k < 3$. $k = \pm 3$ puts the point on the circle; $k < 3$ lets $k = -5$, which is outside.

4. The latus rectum of a parabola with vertex at the origin has end points $(-4, 8)$ and $(-4, -8)$. The equation of the parabola is [1]

(a) $y^2 = 16x$ (b) $y^2 = -16x$

(c) $x^2 = -16y$ (d) $y^2 = -8x$

Answer: (b) The latus rectum passes through the focus and is perpendicular to the axis, so the focus is its mid-point $(-4, 0)$ and the axis is the x-axis. Then $a = 4$ and the parabola opens to the left: $y^2 = -16x$. Check: the latus rectum is $16 = 4a$.

5. A plane passes through the vertex of a double-napped cone and contains one of its generators, so that $\beta = \alpha$. The section is [1]

(a) a point (b) a pair of intersecting straight lines

(c) a circle (d) a straight line

Answer: (d) At the vertex with $\beta = \alpha$ the plane contains a generator and the section is a straight line, the degenerated case of a parabola. $\alpha < \beta \le 90°$ gives a point; $0 \le \beta < \alpha$ a pair of intersecting lines.

6. Find the points of the parabola $x^2 = 8y$ whose distance from the focus is 10 units. [2]

$4a = 8$, so $a = 2$: the focus is $(0, 2)$ and the directrix $y = -2$. By the definition, the distance from the focus equals the distance from the directrix, $y + 2 = 10$, so $y = 8$. Then $x^2 = 64$ and $x = \pm 8$: the points are $(8, 8)$ and $(-8, 8)$.

- focus $(0, 2)$, directrix $y = -2$ and the equation $y + 2 = 10$ (1)
- $y = 8$ and the points $(\pm 8, 8)$ (1)

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7. The eccentricity of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b)$ is $\frac{1}{2}$ and its minor axis is $6\sqrt{3}$ units long. Find a and write the equation of the ellipse. [2]

$b = 3\sqrt{3}$, so $b^2 = 27$. $b^2 = a^2 - c^2 = a^2(1 - e^2) = \frac{3}{4}a^2$, so $a^2 = 36$ and $a = 6$. The ellipse is $\frac{x^2}{36} + \frac{y^2}{27} = 1$.

- $b^2 = a^2(1 - e^2)$ with $b^2 = 27$ (1)
- $a = 6$ and $\frac{x^2}{36} + \frac{y^2}{27} = 1$ (1)

8. Show that the equation $x^2 + y^2 + 4x - 6y + 15 = 0$ does not represent a circle. [2]

Completing the squares: $(x^2 + 4x + 4) + (y^2 - 6y + 9) = -15 + 4 + 9$, i.e. $(x + 2)^2 + (y - 3)^2 = -2$. A sum of two squares cannot be negative, so no point (x, y) satisfies the equation, and it is not a circle ($g^2 + f^2 - c = 4 + 9 - 15 = -2 < 0$).

- completes the squares to $(x + 2)^2 + (y - 3)^2 = -2$ (1)
- concludes no real point satisfies it, so it is not a circle (1)

9. A point P moves so that the difference of its distances from A(0, −3) and B(0, 3) is always 4. Show, from this condition, that the locus of P is a hyperbola, and write its equation in standard form. [3]

$\sqrt{(x^2 + (y + 3)^2)} - \sqrt{(x^2 + (y - 3)^2)} = \pm 4$. Then $\sqrt{(x^2 + (y + 3)^2)} = \pm 4 + \sqrt{(x^2 + (y - 3)^2)}$; squaring, $12y - 16 = \pm 8\sqrt{(x^2 + (y - 3)^2)}$, i.e. $3y - 4 = \pm 2\sqrt{(x^2 + (y - 3)^2)}$. Squaring again, $9y^2 - 24y + 16 = 4x^2 + 4y^2 - 24y + 36$, so $5y^2 - 4x^2 = 20$, i.e. $\frac{y^2}{4} - \frac{x^2}{5} = 1$: a hyperbola with foci $(0, \pm 3)$, $a = 2$ and $b^2 = 9 - 4 = 5$.

- writes the distance condition and isolates one root (1)
- squares twice correctly (1)
- $\frac{y^2}{4} - \frac{x^2}{5} = 1$ (1)

10. A(−1, 2) and B(5, −6) are the ends of a diameter of a circle. Find the equation of the circle, and decide whether the point (6, 1) lies inside, on or outside it. [3]

The centre is the mid-point $(2, -2)$ of AB, and $AB = \sqrt{(36 + 64)} = 10$, so $r = 5$. The circle is $(x - 2)^2 + (y + 2)^2 = 25$, i.e. $x^2 + y^2 - 4x + 4y - 17 = 0$. For (6, 1): $(6 - 2)^2 + (1 + 2)^2 = 16 + 9 = 25 = r^2$, so the point lies on the circle.

- centre $(2, -2)$ and radius 5 (1)
- $(x - 2)^2 + (y + 2)^2 = 25$ (1)
- $(6 - 2)^2 + (1 + 2)^2 = 25$: on the circle (1)

11. The triangle formed by the vertex of the parabola $y^2 = 4ax$ $(a > 0)$ and the two ends of its latus rectum has area 50 square units. Find the equation of the parabola and of its directrix. [3]

The ends of the latus rectum are $(a, 2a)$ and $(a, -2a)$: base $4a$, at a distance a from the vertex O. Area $= \frac{1}{2} \times 4a \times a = 2a^2 = 50$, so $a = 5$. The parabola is $y^2 = 20x$ and the directrix $x = -5$.

- ends $(a, \pm 2a)$ and area $= 2a^2$ (1)
- $a = 5$ (1)
- $y^2 = 20x$ and directrix $x = -5$ (1)

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