Class 11 Mathematics · Chapter 12: Limits and Derivatives

Answer key and marking scheme · 20 marks · about 40 minutes · based on NCERT

Scan for the free Class 11 Mathematics test

1. The value of $\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}$ is [1]

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{8}$ (d) 4

Answer: (b) Write it as $(x^{1/2}-4^{1/2})/(x-4)$. By the remark to Theorem 2 ($n = 1/2$, $a = 4 > 0$) the limit is $(1/2)\cdot 4^{-1/2} = 1/4$.

2. If $\lim_{x\to 2}\frac{x^2+ax-6}{x-2}$ exists (as a real number), then its value is [1]

(a) 5 (b) −5

(c) 1 (d) 4

Answer: (a) The denominator is 0 at $x = 2$, so for the limit to exist the numerator must also vanish there: $4 + 2a - 6 = 0$, $a = 1$. Then $x^2 + x - 6 = (x + 3)(x - 2)$, and the limit is $2 + 3 = 5$.

3. If $\lim_{x\to 2}(x^2+kx+1) = 11$, then the value of k is [1]

(a) 3 (b) 5

(c) −3 (d) 2

Answer: (a) The limit of a polynomial at $x = 2$ is its value there: $4 + 2k + 1 = 11$, so $2k = 6$ and $k = 3$.

4. The value of $\lim_{x\to 2}\frac{x^5-32}{x-2}$ is [1]

(a) 40 (b) 32

(c) 16 (d) 80

Answer: (d) $32 = 2^5$, so by Theorem 2 the limit is $5(2)^4 = 80$.

5. Let $f(x) = x^2 - 1$ for $x \neq 2$ and $f(2) = 0$. Then $\lim_{x\to 2} f(x) - f(2)$ equals [1]

(a) 0 (b) 3

(c) −3 (d) 4

Answer: (b) Near $x = 2$ (but not at 2), $f(x) = x^2 - 1$ approaches 3; the value at 2 is 0. The limit and the value differ: $3 - 0 = 3$.

6. Find the derivative of $x\sqrt{x} + \frac{1}{\sqrt{x}}$ and its value at $x = 1$. [2]

Write it as $x^{3/2} + x^{-1/2}$. By the power rule (valid for real powers), the derivative is $(3/2)x^{1/2} - (1/2)x^{-3/2}$. At $x = 1$: $3/2 - 1/2 = 1$.

- derivative $(3/2)x^{1/2} - (1/2)x^{-3/2}$ (1)
- value 1 at $x = 1$ (1)

Every question original, written from NCERT, checked by a second reviewer · photocopy freely for your class

7. If $y = x^2 \cos x$, show that $x\frac{dy}{dx} = 2y - x^3 \sin x$. [2]

By the product rule, $dy/dx = 2x \cos x + x^2(-\sin x) = 2x \cos x - x^2 \sin x$. Multiplying by x: $x\,dy/dx = 2x^2 \cos x - x^3 \sin x = 2y - x^3 \sin x$.

- $dy/dx = 2x \cos x - x^2 \sin x$ by the product rule (1)
- multiplies by x and identifies 2y (1)

8. Evaluate $\lim_{x\to 0} \frac{\sin 2x + \sin 4x}{x}$. [2]

Split into $(\sin 2x)/x + (\sin 4x)/x = 2\cdot(\sin 2x)/(2x) + 4\cdot(\sin 4x)/(4x)$. As $x \to 0$, $2x \to 0$ and $4x \to 0$, so each ratio tends to 1 and the limit is $2 + 4 = 6$.

- splits and writes each term as $k\cdot(\sin kx)/(kx)$ (1)
- limit 6 (1)

9. Using Theorem 2 on standard limits, evaluate $\lim_{x\to 1} \frac{x^4 + x^2 - 2}{x - 1}$. [3]

Write $x^4 + x^2 - 2 = (x^4 - 1) + (x^2 - 1)$. Then the limit is $\lim (x^4 - 1^4)/(x - 1) + \lim (x^2 - 1^2)/(x - 1) = 4(1)^3 + 2(1)^1 = 4 + 2 = 6$.

- splits the numerator as $(x^4 - 1) + (x^2 - 1)$ (1)
- applies Theorem 2 to each: 4 and 2 (1)
- limit 6 (1)

10. Differentiate $\frac{x^2 + 1}{x^2 - 1}$, $x \neq \pm 1$. [3]

$u = x^2 + 1$, $v = x^2 - 1$, $u' = v' = 2x$. By the quotient rule, $(u'v - uv')/v^2 = [2x(x^2 - 1) - 2x(x^2 + 1)]/(x^2 - 1)^2 = -4x/(x^2 - 1)^2$.

- $u' = v' = 2x$ and the quotient rule set up (1)
- numerator $2x(x^2 - 1) - 2x(x^2 + 1)$ (1)
- $-4x/(x^2 - 1)^2$ (1)

11. Find the derivative of $f(x) = \sqrt{x}$, $x > 0$, from the first principle. [3]

$f'(x) = \lim_{h\to 0} (\sqrt{(x + h)} - \sqrt{x})/h$. Multiply numerator and denominator by $\sqrt{(x + h)} + \sqrt{x}$: the numerator becomes $(x + h) - x = h$, so the quotient is $1/(\sqrt{(x + h)} + \sqrt{x})$ for $h \neq 0$. As $h \to 0$ this tends to $1/(2\sqrt{x})$. So $f'(x) = 1/(2\sqrt{x})$.

- writes the first-principle limit (1)
- rationalises to $1/(\sqrt{(x + h)} + \sqrt{x})$ (1)
- $f'(x) = 1/(2\sqrt{x})$ (1)

Every question original, written from NCERT, checked by a second reviewer · photocopy freely for your class