Class 11 Mathematics · Chapter 13: Statistics

Answer key and marking scheme · 20 marks · about 40 minutes · based on NCERT

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1. The variance of a set of observations is 16. If each observation is halved, the standard deviation of the new observations is [1]

(a) 4 (b) 2
(c) 8 (d) 1

Answer: (b) Multiplying each observation by k = 1/2 multiplies the variance by k^2 = 1/4: new variance 4, so the new standard deviation is √4 = 2.

2. A data set has mean 12 and variance 9. If 7 is added to each observation, the new mean and the new variance are [1]

(a) 19 and 16 (b) 12 and 9
(c) 19 and 9 (d) 19 and 81

Answer: (c) Adding a number to each observation shifts the mean by that number (12 + 7 = 19) and leaves the variance unchanged (9).

3. The sum of the deviations of the observations 3, 8, 10 and 15 from their mean is [1]

(a) 0 (b) 9
(c) 36 (d) 4

Answer: (a) The mean is 36/4 = 9; the deviations are −6, −1, 1, 6, and their sum is 0, as it is for every data about its mean.

4. The mean deviation about the median of the six observations 11, 3, 8, 15, 6, 20 is (correct to two decimal places) [1]

(a) 4.83 (b) 5.80
(c) 9.50 (d) 29

Answer: (a) In order: 3, 6, 8, 11, 15, 20. n is even, so M = (8 + 11)/2 = 9.5. The absolute deviations are 6.5, 3.5, 1.5, 1.5, 5.5, 10.5, sum 29; M.D.(M) = 29/6 = 4.83. Dividing by 5 gives the slip 5.80.

5. The variance of a data set is 6.25. Its standard deviation is [1]

(a) 6.25 (b) 3.125
(c) 39.0625 (d) 2.5

Answer: (d) The standard deviation is the positive square root of the variance: √6.25 = 2.5.

6. Verify that the sum of the deviations of 5, 9, 12, 14 from their mean is zero, and explain why the mean of these deviations cannot serve as a measure of dispersion. [2]

$\bar{x}$ = 40/4 = 10; the deviations are −5, −1, 2, 4, and their sum is 0. Since this happens for every data set about its mean, the mean of the deviations is always 0/n = 0 and tells nothing about scatter.

- mean 10 and the deviations summing to 0 (1)
- the mean of deviations about the mean is always 0, so it is of no use (1)

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7. For 3, 7, 9, 13, 18, show that the mean deviation about the median is less than the mean deviation about the mean. [2]

$\bar{x} = 50/5 = 10$; $|x_i - 10| = 7, 3, 1, 3, 8$, sum 22, so M.D.($\bar{x}$) = 22/5 = 4.4. The median is the 3rd observation, 9; $|x_i - 9| = 6, 2, 0, 4, 9$, sum 21, so M.D.(M) = 21/5 = 4.2 < 4.4.

- M.D.($\bar{x}$) = 4.4 (1)
- M = 9, M.D.(M) = 4.2 and the comparison (1)

8. Find the range of 18, 25, 9, 31, 14, 22, and state one thing about the data that the range does not tell. [2]

Range = 31 − 9 = 22. The range depends only on the two extreme values; it does not tell how the observations are spread about a central value such as the mean, for which a measure based on the deviations from it is needed.

- range 22 (1)
- says the range ignores how the values lie about a central value (1)

9. Set P is 18, 30, 42. Set Q is the 2m + 1 consecutive integers 30 − m, 30 − m + 1, ..., 30 + m; both sets have mean 30. (i) Find the least m for which $\Sigma (x_i - 30)^2$ for Q exceeds that for P. (ii) For this m, find the variance of each set, and say what the two results show about $\Sigma (x_i - \bar{x})^2$ as a measure of dispersion. [3]

(i) P: deviations −12, 0, 12, so Σ = 144 + 0 + 144 = 288. Q: deviations −m, ..., m, so $\Sigma = 2(1^2 + 2^2 + \ldots + m^2) = m(m + 1)(2m + 1)/3$. For m = 7 this is 7 × 8 × 15/3 = 280 < 288; for m = 8 it is 8 × 9 × 17/3 = 408 > 288. The least m is 8 (Q is 22, 23, ..., 38, seventeen integers). (ii) Variance of P = 288/3 = 96; variance of Q = 408/17 = 24. Q has the larger sum although P's values lie far wider of the mean: the sum grows with the number of observations, so the mean of the squared deviations, the variance, is the proper measure.

- P's sum 288 and Q's sum m(m + 1)(2m + 1)/3 (1)
- m = 8 (m = 7 gives 280), Q's sum 408 (1)
- variances 96 and 24, and the sum rejected as a measure, with reason (1)

10. Find the standard deviation of x_i: 2, 4, 6, 8, 10 with f_i: 1, 4, 6, 4, 1. What would the standard deviation be if every frequency were doubled? [3]

N = 16, $\Sigma f_i x_i = 2 + 16 + 36 + 32 + 10 = 96$, $\bar{x} = 6$. $(x_i - 6)^2 = 16, 4, 0, 4, 16$; $\Sigma f_i (x_i - \bar{x})^2 = 16 + 16 + 0 + 16 + 16 = 64$; $\sigma^2 = 64/16 = 4$, $\sigma = 2$. Doubling every frequency doubles both $\Sigma f_i (x_i - \bar{x})^2$ and N, and leaves the mean at 6, so σ is still 2.

- $\bar{x} = 6$ (1)
- $\sigma^2 = 64/16 = 4$, $\sigma = 2$ (1)
- doubling frequencies leaves σ = 2, with reason (1)

11. For the distribution x_i: 10, 20, 30, 40, 50 with frequencies f_i: 4, 7, 12, 9, k, the median is 30 and the mean deviation about the median is 10. Find k. [3]

$|x_i - 30| = 20, 10, 0, 10, 20$, so $\Sigma f_i |x_i - M| = 80 + 70 + 0 + 90 + 20k = 240 + 20k$ and N = 32 + k. (240 + 20k)/(32 + k) = 10 gives 240 + 20k = 320 + 10k, so k = 8. Check: N = 40, cumulative frequencies 4, 11, 23, 32, 40; the 20th and 21st observations are 30, so the median is indeed 30.

- $\Sigma f_i |x_i - 30| = 240 + 20k$ and N = 32 + k (1)
- equation (240 + 20k)/(32 + k) = 10 (1)
- k = 8, with the median checked (1)