Class 11 Mathematics · Chapter 2: Relations and Functions

Answer key and marking scheme · 20 marks · about 40 minutes · based on NCERT

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1. The number of relations from A to B is 4096. If n(A) = 4, then n(B) is [1]

(a) 3 (b) 8

(c) 4 (d) 1024

Answer: (a) The number of relations is $2^{n(A)\,n(B)} = 2^{4\,n(B)} = 4096 = 2^{12}$, so 4 n(B) = 12 and n(B) = 3. 1024 divides 4096 by 4 without taking the logarithm.

2. If A = {1, 2} and B = {2, 3}, then (A × B) ∩ (B × A) is [1]

(a) ϕ (b) {(1, 2), (2, 1)}

(c) {(2, 3), (3, 2)} (d) {(2, 2)}

Answer: (d) A × B = {(1, 2), (1, 3), (2, 2), (2, 3)} and B × A = {(2, 1), (2, 2), (3, 1), (3, 2)}. Order matters, so (1, 2) and (2, 1) are different pairs; the only pair in both is (2, 2).

3. The rule f(x) = x + k for −4 ≤ x ≤ 1 and f(x) = 3x − 1 for 1 ≤ x ≤ 6 defines a function on [−4, 6] only when k equals [1]

(a) 2 (b) 0

(c) 1 (d) −1

Answer: (c) x = 1 lies in both pieces, so its two values must agree: 1 + k = 3 × 1 − 1 = 2, giving k = 1. For any other k the element 1 has two images and the rule is not a function.

4. If n(A × B) = 15 and n(A) = 5, then n(B) is [1]

(a) 10 (b) 3

(c) 20 (d) 75

Answer: (b) n(A × B) = n(A) × n(B), so 15 = 5 × n(B) and n(B) = 3. 10 subtracts, 20 adds and 75 multiplies the two given numbers.

5. The number of relations on a set A having 3 elements is [1]

(a) 9 (b) 512

(c) 64 (d) 8

Answer: (b) A relation on A is a subset of A × A, which has 3 × 3 = 9 elements, so there are $2^9 = 512$ relations. 9 counts the pairs, not the subsets; $8 = 2^3$ counts the subsets of A.

6. Let A = {0, 1}. List the ordered triplets in A × A × A that have exactly two entries equal to 1. [2]

A × A × A has 2 × 2 × 2 = 8 triplets. Those with exactly two 1s have a single 0, in the first, second or third place: (0, 1, 1), (1, 0, 1), (1, 1, 0). There are 3; order matters, so they are different triplets.

- uses ordered triplets (a, b, c) with a, b, c ∈ A (1)
- lists (0, 1, 1), (1, 0, 1), (1, 1, 0) (1)

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7. A relation R on N is R = {(x, y) : x + 2y = 10, x, y ∈ N}. Explain why (0, 5) ∉ R, and show that every element of the domain of R has exactly one image. [2]

0 is not a natural number, so (0, 5) is not in N × N and cannot be in R, though 0 + 2 × 5 = 10. For y = 1, 2, 3, 4, x = 10 − 2y = 8, 6, 4, 2, so R = {(8, 1), (6, 2), (4, 3), (2, 4)} with domain {2, 4, 6, 8}. Each x in the domain gives the single value y = (10 − x)/2, so it has exactly one image.

- (0, 5) fails because 0 ∉ N (1)
- domain {2, 4, 6, 8} and y = (10 − x)/2 unique for each x (1)

8. f : R → R is a constant function with f(3) = −2. Find f(−7) + f(0) and the range of f. [2]

A constant function takes the same value at every x, so f(x) = −2 for all x. f(−7) + f(0) = −2 + (−2) = −4. The range is {−2}.

- f(x) = −2 for every x, so the sum is −4 (1)
- range {−2} (1)

9. Let f(x) = √(x + 2) and g(x) = x^2 − 1. Find the domain of f + g and of $\frac{f}{g}$, and find $(\frac{f}{g})(2)$. [3]

f is defined for x ≥ −2, g for all real x. f + g is defined on [−2, ∞). f/g also needs g(x) ≠ 0, i.e. x ≠ 1 and x ≠ −1, both in [−2, ∞): domain [−2, ∞) − {−1, 1}. (f/g)(2) = √4 / (4 − 1) = 2/3.

- domain of f + g: [−2, ∞) (1)
- domain of f/g: [−2, ∞) − {−1, 1} (1)
- (f/g)(2) = 2/3 (1)

10. A × B = {(x, 1), (x, 3), (y, 1), (y, 3)} and A × C = {(x, 3), (x, 5), (y, 3), (y, 5)}. Find A, B and C, and hence write A × (B ∩ C). Check that it equals (A × B) ∩ (A × C). [3]

The first elements of A × B give A = {x, y}; its second elements give B = {1, 3}. From A × C, C = {3, 5}. B ∩ C = {3}, so A × (B ∩ C) = {(x, 3), (y, 3)}. The pairs common to A × B and A × C are (x, 3) and (y, 3), the same set.

- A = {x, y}, B = {1, 3}, C = {3, 5} (1)
- B ∩ C = {3} and A × (B ∩ C) = {(x, 3), (y, 3)} (1)
- (A × B) ∩ (A × C) = {(x, 3), (y, 3)}, equal (1)

11. Let A = {1, 2} and B = {3, 5}. (i) Write every function from A to B in roster form. (ii) Find the number of relations from A to B that are not functions from A to B. (iii) Find the number of relations from A to B whose domain is A. [3]

A × B = {(1, 3), (1, 5), (2, 3), (2, 5)}. (i) A function from A to B gives each of 1 and 2 exactly one image, 3 or 5: {(1, 3), (2, 3)}, {(1, 3), (2, 5)}, {(1, 5), (2, 3)}, {(1, 5), (2, 5)}, four in all. (ii) A relation is any subset of A × B, so there are 2^4 = 16 relations; 16 − 4 = 12 of them are not functions. (iii) The domain is A when 1 and 2 each have at least one image. The pairs with first element 1 can be {(1, 3)}, {(1, 5)} or both: 3 ways; likewise 3 ways for 2. So 3 × 3 = 9 relations have domain A; 4 of them are the functions of (i), and in the other 5 some element has two images.

- (i) the four functions listed (1)
- (ii) 2^4 − 4 = 12 (1)
- (iii) 3 × 3 = 9 (1)

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