Class 11 Mathematics · Chapter 3: Trigonometric Functions

Answer key and marking scheme · 20 marks · about 40 minutes · based on NCERT

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1. Which of the following is equal to sin 20°? [1]

(a) sin 380° (b) sin (−20°)

(c) sin 200° (d) cos 20°

Answer: (a) 380° = 360° + 20°, and sin (2nπ + x) = sin x, so sin 380° = sin 20°. sin (−20°) = −sin 20°, sin 200° = sin (180° + 20°) = −sin 20°, and cos 20° = sin 70°.

2. Which of the following can never be a value of cos x for a real number x? [1]

(a) −1 (b) 3/4

(c) 1/√2 (d) π/3

Answer: (d) For every real x, −1 ≤ cos x ≤ 1. π/3 ≈ 1.047 is greater than 1; the other three lie in [−1, 1] (cos π = −1).

3. If $\tan x = \frac{1}{2}$, then sin 2x is [1]

(a) $\frac{3}{5}$ (b) $\frac{4}{3}$

(c) 1 (d) $\frac{4}{5}$

Answer: (d) sin 2x = 2 tan x/(1 + $\tan^2$ x) = 1/(1 + 1/4) = 4/5. 4/3 is tan 2x and 3/5 is cos 2x.

4. If sin x > 0 and tan x < 0, then x lies in the [1]

(a) first quadrant (b) second quadrant

(c) third quadrant (d) fourth quadrant

Answer: (b) sin x is positive in quadrants I and II; tan x is negative in quadrants II and IV. Both hold only in the second quadrant.

5. The radian measure of 135° is [1]

(a) 2π/3 (b) 3π/4

(c) 5π/6 (d) 3π/5

Answer: (b) Radian measure = (π/180) × 135 = 135π/180 = 3π/4. 2π/3 is 120° and 5π/6 is 150°.

6. Without finding their values, arrange sin 1, sin 2 and sin 3 (the angles in radians) in increasing order. Give a reason. [2]

Take π ≈ 3.14. sin 2 = sin (π − 2) = sin 1.14 and sin 3 = sin (π − 3) = sin 0.14. The numbers 0.14, 1 and 1.14 all lie between 0 and π/2 ≈ 1.57, where sin x increases as x increases. As 0.14 < 1 < 1.14, sin 3 < sin 1 < sin 2.

- rewrites sin 2 and sin 3 as sin (π − 2) and sin (π − 3), angles in (0, π/2) (1)
- uses that sin x increases on (0, π/2) to give sin 3 < sin 1 < sin 2 (1)

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7. Write the domain of $y = \tan x$ and of $y = \cot x$. Is there a real number at which both $\tan x$ and $\cot x$ are undefined? Justify. [2]

$\tan x = \sin x/\cos x$ needs $\cos x \neq 0$: domain $\{x : x \in R, x \neq (2n + 1)\pi/2, n \in Z\}$. $\cot x = \cos x/\sin x$ needs $\sin x \neq 0$: domain $\{x : x \in R, x \neq n\pi, n \in Z\}$. A number in neither domain must be both an odd multiple of $\pi/2$ and a multiple of π, which is impossible: an odd multiple of $\pi/2$ has $\cos x = 0$ and so $\sin x = \pm 1 \neq 0$. So there is no such number; at every real x at least one of $\tan x$ and $\cot x$ is defined.

- writes both domains correctly (1)
- answers no, with the reason that $\cos x$ and $\sin x$ are never both 0 (1)

8. In a circle, an arc of length 11 cm subtends an angle of 30° at the centre. Find the length of an arc of the same circle that subtends an angle of 1.5 radian at the centre. (Use $\pi = 22/7$.) [2]

30° = $\pi/6$ radian, so $r = l/\theta = 11/(\pi/6) = 66/\pi = 66 \times 7/22 = 21$ cm. The second arc is $l = r\theta = 21 \times 1.5 = 31.5$ cm.

- converts 30° to $\pi/6$ and finds $r = 21$ cm (1)
- arc $= 21 \times 1.5 = 31.5$ cm (1)

9. Find the range of $y = 3 - 2 \sin x$. Find the values of x in $[0, 2\pi]$ at which y takes its greatest and its least value. [3]

$-1 \leq \sin x \leq 1$, so $-2 \leq -2 \sin x \leq 2$ (multiplying by -2 reverses the inequalities) and $1 \leq 3 - 2 \sin x \leq 5$. Every value in [1, 5] is taken, as $\sin x$ takes every value in $[-1, 1]$; the range is [1, 5]. The greatest value 5 comes when $\sin x = -1$, at $x = 3\pi/2$; the least value 1 when $\sin x = 1$, at $x = \pi/2$.

- uses $-1 \leq \sin x \leq 1$ and reverses the inequality for -2 (1)
- range [1, 5] (1)
- greatest at $x = 3\pi/2$, least at $x = \pi/2$ (1)

10. If $\tan x = \frac{1}{2}$ and $\tan y = \frac{1}{3}$, where x and y are acute angles, show that $x + y = \pi/4$. [3]

$\tan (x + y) = (\tan x + \tan y)/(1 - \tan x \tan y) = (1/2 + 1/3)/(1 - 1/6) = (5/6)/(5/6) = 1$. As x and y are acute, $0 < x + y < \pi$. In $(0, \pi)$ the only angle with tangent 1 is $\pi/4$ ($\tan x$ is negative in $(\pi/2, \pi)$ and undefined at $\pi/2$). So $x + y = \pi/4$.

- applies $\tan (x + y)$ correctly (1)
- gets $\tan (x + y) = 1$ (1)
- uses $0 < x + y < \pi$ to conclude $x + y = \pi/4$ (1)

11. Without finding the value of $\tan 15°$ or of $\tan 75°$, show that $\tan 15° + \tan 75° = 4$. [3]

$\tan A + \tan B = \sin A/\cos A + \sin B/\cos B = (\sin A \cos B + \cos A \sin B)/(\cos A \cos B) = \sin (A + B)/(\cos A \cos B)$. With $A = 15°$ and $B = 75°$: $\sin (A + B) = \sin 90° = 1$, and $\cos 75° = \cos (90° - 15°) = \sin 15°$. So $\tan 15° + \tan 75° = 1/(\cos 15° \sin 15°) = 2/(2 \sin 15° \cos 15°) = 2/\sin 30° = 2/(1/2) = 4$.

- writes $\tan A + \tan B = \sin (A + B)/(\cos A \cos B)$ (1)
- uses $\sin 90° = 1$ and $\cos 75° = \sin 15°$ (1)
- uses $2 \sin 15° \cos 15° = \sin 30° = 1/2$ to get 4 (1)

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