Class 11 Mathematics · Chapter 4: Complex Numbers and Quadratic Equations

Answer key and marking scheme · 20 marks · about 40 minutes · based on NCERT

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1. The real number k for which $\frac{2 + ki}{1 - 2i}$ is purely real is [1]

(a) −4 (b) 4

(c) 1 (d) −1

Answer: (a) Multiply above and below by the conjugate 1 + 2i: (2 + ki)(1 + 2i)/5 = [(2 − 2k) + (4 + k)i]/5. It is purely real when its imaginary part is 0, so 4 + k = 0 and k = −4. Taking 4 drops the sign of $i^2 = -1$; 1 comes from setting the real part to zero instead.

2. Which of the following is equal to −1? [1]

(a) i^6 (b) i^8

(c) i^{-4} (d) i^9

Answer: (a) 6 = 4 × 1 + 2, so $i^6 = i^2 = -1$. Since 8 and −4 are multiples of 4, $i^8 = i^{-4} = 1$; and 9 = 4 × 2 + 1 gives $i^9 = i$.

3. For the complex number $z = -\sqrt{5} - 8i$, Im z is [1]

(a) −8i (b) 8

(c) −8 (d) $-\sqrt{5}$

Answer: (c) For z = a + ib the imaginary part is the real number b, not ib. Here b = −8, so Im z = −8. Writing −8i takes ib for b; $-\sqrt{5}$ is Re z.

4. In the Argand plane, the point representing the conjugate of z = −3 + 2i lies in the [1]

(a) first quadrant (b) second quadrant

(c) third quadrant (d) fourth quadrant

Answer: (c) $\bar{z} = -3 - 2i$ is the point (−3, −2), the mirror image of (−3, 2) in the real axis; both coordinates are negative, so it lies in the third quadrant. The second quadrant is where z itself lies.

5. The value of $i + i^2 + i^3 + \ldots + i^{50}$ is [1]

(a) 0 (b) i

(c) −1 − i (d) −1 + i

Answer: (d) Every four consecutive powers of i add to 0, so the first 48 terms add to 0. What is left is $i^{49} + i^{50} = i + (-1) = -1 + i$. The answer 0 forgets that 50 is not a multiple of 4.

6. Find all complex numbers z for which $\bar{z} = iz$ and $|z| = \sqrt{2}$. [2]

Let z = a + ib. Then iz = −b + ia, and $\bar{z} = a - ib$. Equating parts: a = −b and −b = a, so b = −a and z = a − ia. Then $|z| = \sqrt{(a^2 + a^2)} = |a|\sqrt{2} = \sqrt{2}$, so a = ±1. Hence z = 1 − i or z = −1 + i.

- writes iz = −b + ia and equates with a − ib to get b = −a (1)
- uses $|z| = \sqrt{2}$ to get a = ±1: z = 1 − i or −1 + i (1)

7. Taking a = −4 and b = −25, show that $\sqrt{a} \times \sqrt{b} \neq \sqrt{(ab)}$. [2]

$\sqrt{(-4)} = 2i$ and $\sqrt{(-25)} = 5i$, so $\sqrt{a} \times \sqrt{b} = 10i^2 = -10$. But ab = 100 and $\sqrt{(ab)} = 10$. Since −10 ≠ 10, $\sqrt{a} \times \sqrt{b} \neq \sqrt{(ab)}$ when a and b are both negative.

- finds $\sqrt{a} \times \sqrt{b} = (2i)(5i) = -10$ (1)
- finds $\sqrt{(ab)} = 10$ and concludes they differ (1)

8. Write $z = \frac{2 + i}{1 - i}$ in the form a + ib. In which quadrant of the Argand plane does z lie, and how far is it from the origin? [2]

$z = (2 + i)(1 + i)/((1 - i)(1 + i)) = (2 + 2i + i - 1)/2 = 1/2 + (3/2)i$. The point (1/2, 3/2) has both coordinates positive, so it is in the first quadrant. Its distance from O is $|z| = \sqrt{(1/4 + 9/4)} = \sqrt{10}/2$.

- $z = 1/2 + (3/2)i$ (1)
- first quadrant and distance $\sqrt{10}/2$ (1)

9. If x and y are real and $(x + iy)^2 = 7 + 24i$, find $x^2 + y^2$, and hence find all possible pairs (x, y). [3]

Taking moduli, $|x + iy|^2 = |7 + 24i|$, so $x^2 + y^2 = \sqrt{(49 + 576)} = 25$. Expanding, $(x + iy)^2 = (x^2 - y^2) + 2xyi$, so $x^2 - y^2 = 7$ and $2xy = 24$. Adding and subtracting with $x^2 + y^2 = 25$: $x^2 = 16$, $y^2 = 9$. Since $xy = 12 > 0$, x and y have the same sign: (x, y) = (4, 3) or (−4, −3).

- $x^2 + y^2 = |7 + 24i| = 25$ (1)
- $x^2 - y^2 = 7$, $2xy = 24$, so $x^2 = 16$, $y^2 = 9$ (1)
- uses $xy > 0$: (4, 3) or (−4, −3) (1)

10. For $z_1 = 1 + 2i$ and $z_2 = 3 - i$, show that $z_1 \bar{z}_2 + \bar{z}_1 z_2$ is a real number, and find it. [3]

$\bar{z}_1 z_2$ is the conjugate of $z_1 \bar{z}_2$ (the conjugate of a product is the product of the conjugates). So the sum is $w + \bar{w} = 2 \operatorname{Re} w$ with $w = z_1 \bar{z}_2$, which is real. $w = (1 + 2i)(3 + i) = 3 + i + 6i + 2i^2 = 1 + 7i$, so the sum is $2 \times 1 = 2$. Check: $\bar{z}_1 z_2 = (1 - 2i)(3 - i) = 1 - 7i$, and $(1 + 7i) + (1 - 7i) = 2$.

- notes $\bar{z}_1 z_2$ is the conjugate of $z_1 \bar{z}_2$, so the sum is $2 \operatorname{Re}(z_1 \bar{z}_2)$, real (1)
- finds $z_1 \bar{z}_2 = 1 + 7i$ (1)
- the sum is 2 (1)

11. Let $z_1 = 2 + i$ and $z_2 = 2 - i$. Using the identity $(z_1 + z_2)^3 = z_1^3 + z_2^3 + 3z_1z_2(z_1 + z_2)$, find $z_1^3 + z_2^3$. Why must the answer be real? [3]

$z_1 + z_2 = 4$ and $z_1z_2 = |z_1|^2 = 5$. So $4^3 = z_1^3 + z_2^3 + 3 \times 5 \times 4$, that is $64 = z_1^3 + z_2^3 + 60$, and $z_1^3 + z_2^3 = 4$. It is real because $z_2 = \bar{z}_1$, so z_2^3 is the conjugate of z_1^3 and the sum is $2 \operatorname{Re}(z_1^3)$. (Check: $z_1^3 = 2 + 11i$.)

- $z_1 + z_2 = 4$ and $z_1z_2 = 5$ (1)
- substitutes in the identity: $z_1^3 + z_2^3 = 4$ (1)
- reason: z_2^3 is the conjugate of z_1^3, so the sum is $2 \operatorname{Re} z_1^3$ (1)