Class 11 Mathematics · Chapter 6: Permutations and Combinations

Answer key and marking scheme · 20 marks · about 40 minutes · based on NCERT

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1. The value of ${}^{8}P_{0} + {}^{8}C_{8} + 0!$ is [1]

(a) 1 (b) 2

(c) 3 (d) 0

Answer: (c) ${}^{n}P_{0} = 1$, ${}^{n}C_{n} = 1$ and $0! = 1$, so the sum is $1 + 1 + 1 = 3$.

2. The value of ${}^{15}C_{13}$ is [1]

(a) 105 (b) 210

(c) 2730 (d) 15

Answer: (a) ${}^{15}C_{13} = {}^{15}C_{2} = \frac{15 \times 14}{2} = 105$. 210 is ${}^{15}P_{2}$, the count without dividing by 2!.

3. A word has 6 letters, of which one letter appears exactly twice and the other four are all different. [1] The number of arrangements of all its letters is

(a) 720 (b) 120

(c) 180 (d) 360

Answer: (d) By Theorem 3, $n!/p! = 6!/2! = 720/2 = 360$.

4. Seven runners take part in a school race. In how many ways can the gold, silver and bronze medals [1] be awarded, if no runner gets more than one medal?

(a) 210 (b) 35

(c) 343 (d) 5040

Answer: (a) The medals are different, so order matters: ${}^{7}P_{3} = 7 \times 6 \times 5 = 210$. ${}^{7}C_{3} = 35$ ignores which medal goes to whom; 7^{3} lets one runner win twice.

5. Five different letters are to be posted, each in any one of 3 letter boxes. The number of ways of [1] posting them is

(a) 125 (b) 15

(c) 243 (d) 60

Answer: (c) Each of the 5 letters has 3 choices of box, and a box may take any number of letters, so the count is $3 \times 3 \times 3 \times 3 \times 3 = 3^{5} = 243$. 5^{3} gives each box a choice of letter, which is the wrong way round.

6. Write the product $1 \times 3 \times 5 \times 7 \times 9$ in terms of factorials. [2]

$10! = (1 \times 3 \times 5 \times 7 \times 9) \times (2 \times 4 \times 6 \times 8 \times 10)$, and $2 \times 4 \times 6 \times 8 \times 10 = 2^{5} \times (1 \times 2 \times 3 \times 4 \times 5) = 2^{5} \times 5!$. So $1 \times 3 \times 5 \times 7 \times 9 = \frac{10!}{2^{5} \times 5!} = 3628800/3840 = 945$.

- splits 10! into the odd and the even factors (1)
- writes the even product as $2^{5} \times 5!$ and gives $10!/(2^{5} \times 5!)$ (1)

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7. A team's selectors are to pick 2 spinners from 6 available spinners. In how many ways can they do this? Give a reason why permutations are not used. [2]

The pair of spinners has no order: picking X and then Y gives the same pair as Y and then X. So the count is ${}^{6}C_{2} = \frac{6 \times 5}{2} = 15$.

- states that order does not matter in a selection (1)
- ${}^{6}C_{2} = 15$ (1)

8. Find n if $(n + 1)! = 12 \times (n - 1)!$. [2]

$(n + 1)! = (n + 1) \times n \times (n - 1)!$. Dividing by $(n - 1)!$: $n(n + 1) = 12$, so $n^2 + n - 12 = 0$, $(n + 4)(n - 3) = 0$. As n is a natural number, $n = 3$. Check: $4! = 24 = 12 \times 2!$.

- writes $(n + 1)! = (n + 1)n(n - 1)!$ and gets $n(n + 1) = 12$ (1)
- solves and rejects $n = -4$ to get $n = 3$ (1)

9. A 4-digit PIN is made from the digits 0 to 9; 0 may come first. Find the number of PINs (i) in all, (ii) with all four digits different, (iii) with at least one digit repeated. [3]

(i) Each place has 10 choices: $10^4 = 10000$. (ii) No repetition: ${}^{10}P_{4} = 10 \times 9 \times 8 \times 7 = 5040$. (iii) At least one repeat is the complement of (ii): $10000 - 5040 = 4960$.

- (i) $10^4 = 10000$ (1)
- (ii) ${}^{10}P_{4} = 5040$ (1)
- (iii) $10000 - 5040 = 4960$ (1)

10. A team of 4 is chosen from 7 volunteers, and one member of the team is then made its leader. Find the number of ways of doing this by two methods, and show that the two answers agree. [3]

Method 1: choose the team, ${}^{7}C_{4} = 35$ ways, then its leader, 4 ways: $35 \times 4 = 140$. Method 2: choose the leader first, 7 ways, then the other 3 members from the remaining 6, ${}^{6}C_{3} = 20$ ways: $7 \times 20 = 140$. Both give 140.

- method 1: ${}^{7}C_{4} \times 4 = 140$ (1)
- method 2: $7 \times {}^{6}C_{3} = 140$ (1)
- uses the multiplication principle and states they agree (1)

11. Ten points lie in a plane, of which exactly 4 are collinear; no other three of them are collinear. Find the number of (i) straight lines, (ii) triangles, that can be drawn through these points. [3]

(i) Pairs of points: ${}^{10}C_{2} = 45$. The 4 collinear points give ${}^{4}C_{2} = 6$ pairs but only 1 line, so lines $= 45 - 6 + 1 = 40$. (ii) Sets of 3 points: ${}^{10}C_{3} = 120$; the ${}^{4}C_{3} = 4$ sets of collinear points make no triangle, so triangles $= 120 - 4 = 116$.

- ${}^{4}C_{2}$ pairs on one line counted once (1)
- lines $= 45 - 6 + 1 = 40$ (1)
- triangles $= 120 - 4 = 116$ (1)

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