Class 12 Mathematics · Chapter 1: Relations and Functions

Answer key and marking scheme · 20 marks · about 40 minutes · based on NCERT

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1. The number of reflexive relations in a set with 3 elements is [1]

(a) 8 (b) 512

(c) 64 (d) 27

Answer: (c) A × A has 9 pairs. A reflexive relation must contain the 3 pairs (a, a); each of the other 9 − 3 = 6 pairs may be in or out, giving $2^6 = 64$. $512 = 2^9$ counts all relations.

2. Let A = {2, 4} and B = {1, 3, 5}. The number of relations from A to B is [1]

(a) 6 (b) 64

(c) 32 (d) 8

Answer: (b) A relation from A to B is any subset of A × B. A × B has 2 × 3 = 6 elements, so it has $2^6 = 64$ subsets. 6 counts the pairs, not the subsets; 32 and 8 are 2^5 and 2^3.

3. In the set A = {1, 2, 3, 4}, the relation R = {(a, b) : $|a^2 - b^2| < 16$} is [1]

(a) the empty relation (b) the universal relation

(c) reflexive but not symmetric (d) symmetric but not reflexive

Answer: (b) The squares of the elements are 1, 4, 9, 16, so the largest value of $|a^2 - b^2|$ is 16 − 1 = 15 < 16. Every pair of A × A is in R, so R = A × A, the universal relation (which is also reflexive and symmetric, so (c) and (d) are false).

4. The number of relations in the set {1, 2, 3} that are both reflexive and symmetric is [1]

(a) 6 (b) 64

(c) 8 (d) 512

Answer: (c) The three pairs (1, 1), (2, 2), (3, 3) must be present. Symmetry makes the other six pairs come in three couples {(1, 2), (2, 1)}, {(1, 3), (3, 1)}, {(2, 3), (3, 2)}, each wholly in or wholly out: $2^3 = 8$. 64 counts reflexive relations without the symmetry condition.

5. Let X = {1, 2} and let R be the relation in the power set P(X) given by A R B if and only if A ⊂ B (A is a subset of B, A = B allowed). The number of ordered pairs in R is [1]

(a) 5 (b) 16

(c) 9 (d) 4

Answer: (c) P(X) = {ϕ, {1}, {2}, {1, 2}}. Pairs with A ⊂ B: ϕ with all four sets (4), {1} with {1} and {1, 2} (2), {2} with {2} and {1, 2} (2), {1, 2} with itself (1): 4 + 2 + 2 + 1 = 9. Equivalently, each of 1 and 2 is in neither set, in B only, or in both, giving $3^2 = 9$. 16 counts all pairs; 5 leaves out the four pairs (A, A).

6. Show that f : R → R, $f(x) = x^3 + 2$, is invertible, and find $f^{-1}(10)$. [2]

$x_1^3 + 2 = x_2^3 + 2$ gives $x_1^3 = x_2^3$, so $x_1 = x_2$ (x^3 is one-one on R). For any real y, $x = \sqrt[3]{(y - 2)}$ is real and f(x) = y, so f is onto. Being one-one and onto, f is invertible, with $f^{-1}(y) = \sqrt[3]{(y - 2)}$. $f^{-1}(10) = \sqrt[3]{8} = 2$.

- one-one and onto shown, hence invertible (1)
- $f^{-1}(10) = 2$ (1)

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7. Show that $f : R - \{0\} \to R$, $f(x) = \frac{x+3}{x}$, is one-one, and find the one real number that is not the image of any x. [2]

$f(x) = 1 + 3/x$. If $f(x_1) = f(x_2)$, then $3/x_1 = 3/x_2$, so $x_1 = x_2$: f is one-one. For $y \in R$, $y = 1 + 3/x$ gives $x = 3/(y - 1)$, which exists when $y \neq 1$, and $3/(y - 1) \neq 0$. For $y = 1$ we would need $3/x = 0$, impossible. So 1 is the only real number not in the range, and f is not onto.

- one-one: $3/x_1 = 3/x_2$ gives $x_1 = x_2$ (1)
- $x = 3/(y - 1)$; 1 is not an image (1)

8. Let $f : R \to R$, $f(x) = x^2 - 4x$, and let $R = \{(a, b) : f(a) = f(b)\}$ in R, an equivalence relation. Find the equivalence class [1]. [2]

$[1] = \{b \in R : f(b) = f(1)\}$ and $f(1) = 1 - 4 = -3$. So $b^2 - 4b = -3$, $b^2 - 4b + 3 = 0$, $(b - 1)(b - 3) = 0$, $b = 1$ or 3. Hence $[1] = \{1, 3\}$.

- sets up $f(b) = f(1) = -3$ (1)
- $[1] = \{1, 3\}$ (1)

9. Show that $f : R \to R$, $f(x) = 2x + |x|$, is one-one and onto. [3]

$f(x) = 3x$ for $x \geq 0$ and $f(x) = x$ for $x < 0$; $f(x) \geq 0$ when $x \geq 0$ and $f(x) < 0$ when $x < 0$. One-one: if $f(x_1) = f(x_2)$, the common value has one sign, so x_1, x_2 are on the same side of 0; then $3x_1 = 3x_2$ or $x_1 = x_2$, giving $x_1 = x_2$. Onto: for $y \geq 0$, $x = y/3 \geq 0$ and $f(x) = y$; for $y < 0$, $x = y < 0$ and $f(x) = y$. So f is one-one and onto.

- f written piecewise, with the sign of f(x) noted (1)
- one-one by cases (1)
- onto: preimages y/3 and y (1)

10. $A = \{1, 2, 3, 4, 5, 6\}$ is divided into the subsets $\{1, 4\}$, $\{2, 3, 5\}$ and $\{6\}$. Write the equivalence relation R in A whose equivalence classes are these subsets, in roster form, and find the number of ordered pairs in R. [3]

a R b exactly when a and b lie in the same subset. From $\{1, 4\}$: (1, 1), (1, 4), (4, 1), (4, 4). From $\{2, 3, 5\}$: all 9 pairs (2, 2), (2, 3), (2, 5), (3, 2), (3, 3), (3, 5), (5, 2), (5, 3), (5, 5). From $\{6\}$: (6, 6). R is the union of these; it is reflexive (every element is in some subset), symmetric and transitive (pairs never cross subsets). Number of pairs $= 2^2 + 3^2 + 1^2 = 14$.

- rule: related iff in the same subset (1)
- roster form of R (1)
- 14 ordered pairs (1)

11. Let $A = \{1, 2, 3\}$ and $B = \{p, q\}$. Find (i) the number of functions from A to B, (ii) the number of one-one functions from A to B, (iii) the number of onto functions from A to B. [3]

(i) Each of 1, 2, 3 has 2 choices of image: $2^3 = 8$. (ii) Three distinct images are needed but B has only two elements, so there are 0 one-one functions. (iii) A function fails to be onto only when all three images are p or all are q: 2 such functions, so $8 - 2 = 6$ are onto.

- 8 functions (1)
- 0 one-one, with reason (1)
- 6 onto (1)

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