Class 12 Mathematics · Chapter 10: Vector Algebra

Answer key and marking scheme · 20 marks · about 40 minutes · based on NCERT

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1. For P(3, −1, 4) and Q(−2, 5, 1), the scalar component of $\overrightarrow{QP}$ along $\hat{j}$ is [1]

(a) 6 (b) −6

(c) 4 (d) −4

Answer: (b) $\overrightarrow{QP} = \overrightarrow{OP} - \overrightarrow{OQ} = (3+2)\hat{i} + (-1-5)\hat{j} + (4-1)\hat{k} = 5\hat{i} - 6\hat{j} + 3\hat{k}$, so the $\hat{j}$-component is −6. 6 is the component of $\overrightarrow{PQ}$; 4 comes from adding the $\hat{j}$-coordinates.

2. A vector has direction cosines $l = \frac{2}{7}$, $m = -\frac{3}{7}$ and n, where $n < 0$. Then n is [1]

(a) $-\frac{6}{7}$ (b) $\frac{6}{7}$

(c) $-\frac{5}{7}$ (d) $-\frac{1}{7}$

Answer: (a) $l^2 + m^2 + n^2 = 1$ gives $n^2 = 1 - \frac{4}{49} - \frac{9}{49} = \frac{36}{49}$, so $n = \pm\frac{6}{7}$; with $n < 0$, $n = -\frac{6}{7}$. $\frac{6}{7}$ ignores the condition $n < 0$.

3. The vector $\hat{k} \times \hat{i} - \hat{j} \times \hat{k}$ equals [1]

(a) $\hat{i} + \hat{j}$ (b) $\hat{i} - \hat{j}$

(c) $\hat{j} - \hat{i}$ (d) $\vec{0}$

Answer: (c) $\hat{k} \times \hat{i} = \hat{j}$ and $\hat{j} \times \hat{k} = \hat{i}$, so $\hat{k} \times \hat{i} - \hat{j} \times \hat{k} = \hat{j} - \hat{i}$. Reversing either product changes a sign.

4. $\overrightarrow{b}$ is opposite in direction to $\overrightarrow{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ and $|\overrightarrow{b}| = 14$. Then $\overrightarrow{a} \cdot \overrightarrow{b}$ equals [1]

(a) 98 (b) −98

(c) −14 (d) 0

Answer: (b) $|\overrightarrow{a}| = \sqrt{(4 + 9 + 36)} = 7$. Opposite directions mean $\theta = \pi$, so $\overrightarrow{a} \cdot \overrightarrow{b} = |\overrightarrow{a}||\overrightarrow{b}| \cos\pi = -7 \times 14 = -98$. Forgetting $\cos\pi = -1$ gives 98; 0 would need perpendicular vectors.

5. The vectors $2\hat{i} + \lambda\hat{j} - 3\hat{k}$ and $4\hat{i} - \hat{j} + 2\hat{k}$ are perpendicular when λ equals [1]

(a) 2 (b) −2

(c) 14 (d) −14

Answer: (a) Perpendicular means the scalar product is 0: $8 - \lambda - 6 = 0$, so $\lambda = 2$. Adding −6 as +6 gives 14.

6. For vectors $\overrightarrow{a}$ and $\overrightarrow{b}$, $\overrightarrow{a} \cdot \overrightarrow{b} = 6$ and $|\overrightarrow{a} \times \overrightarrow{b}| = 6\sqrt{3}$. If $|\overrightarrow{a}| = 3$, find the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$ and $|\overrightarrow{b}|$. [2]

$|\overrightarrow{a}||\overrightarrow{b}| \sin\theta = 6\sqrt{3}$ and $|\overrightarrow{a}||\overrightarrow{b}| \cos\theta = 6$, so $\tan\theta = \sqrt{3}$ and $\theta = \frac{\pi}{3}$ ($\cos\theta > 0$). Then $3|\overrightarrow{b}| \cos\frac{\pi}{3} = 6$ gives $|\overrightarrow{b}| = 4$.

- divides to get $\tan\theta = \sqrt{3}$, $\theta = \frac{\pi}{3}$ (1)
- $|\overrightarrow{b}| = 4$ (1)

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7. A, B and C have position vectors $\hat{i} + 2\hat{j} - \hat{k}$, $3\hat{i} - \hat{j} + 2\hat{k}$ and $4\hat{i} + \hat{j} + 5\hat{k}$. Find the position vector of D such that ABCD is a parallelogram. [2]

In a parallelogram ABCD, $\vec{AD} = \vec{BC}$, so $\vec{OD} = \vec{OA} + \vec{OC} - \vec{OB} = (1 + 4 - 3)\hat{i} + (2 + 1 + 1)\hat{j} + (-1 + 5 - 2)\hat{k} = 2\hat{i} + 4\hat{j} + 2\hat{k}$. Check: the mid-points of AC and BD are both $\frac{5}{2}\hat{i} + \frac{3}{2}\hat{j} + 2\hat{k}$.

- uses $\vec{AD} = \vec{BC}$ (or diagonals bisecting): $\vec{OD} = \vec{OA} + \vec{OC} - \vec{OB}$ (1)
- $\vec{OD} = 2\hat{i} + 4\hat{j} + 2\hat{k}$ (1)

8. Can a vector make angles of 45°, 60° and 120° with the positive x, y and z-axes? Give a reason. [2]

Direction cosines must satisfy $l^2 + m^2 + n^2 = 1$. Here $\cos^2 45° + \cos^2 60° + \cos^2 120° = \frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1$, so yes; for example the unit vector $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{2}\hat{j} - \frac{1}{2}\hat{k}$ has these direction angles.

- states $l^2 + m^2 + n^2 = 1$ (1)
- computes the sum 1 and concludes yes (1)

9. The projection of $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units. Find λ, and then the projection of $\vec{b}$ on $\vec{a}$. [3]

$|\vec{b}| = \sqrt{(4 + 36 + 9)} = 7$. Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{2\lambda + 6 + 12}{7} = 4$, so $2\lambda + 18 = 28$ and $\lambda = 5$.

Then $\vec{a} = 5\hat{i} + \hat{j} + 4\hat{k}$, $|\vec{a}| = \sqrt{42}$, $\vec{a} \cdot \vec{b} = 28$, and the projection of $\vec{b}$ on $\vec{a}$ is $\frac{28}{\sqrt{42}} = \frac{2\sqrt{42}}{3}$.

- $|\vec{b}| = 7$ and $\frac{2\lambda + 18}{7} = 4$ (1)
- $\lambda = 5$ (1)
- projection of $\vec{b}$ on $\vec{a} = \frac{28}{\sqrt{42}}$ (1)

10. Three vertices of a parallelogram ABCD are A(1, 2, 0), B(3, 1, 2) and D(0, 4, 1). Find the coordinates of C and the area of the parallelogram. [3]

$\vec{AB} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{AD} = -\hat{i} + 2\hat{j} + \hat{k}$. $\vec{OC} = \vec{OB} + \vec{AD} = 2\hat{i} + 3\hat{j} + 3\hat{k}$, so C(2, 3, 3). $\vec{AB} \times \vec{AD} = |\hat{i}\ \hat{j}\ \hat{k};\ 2\ -1\ 2;\ -1\ 2\ 1| = -5\hat{i} - 4\hat{j} + 3\hat{k}$, and the area is $\sqrt{(25 + 16 + 9)} = 5\sqrt{2}$ square units.

- C(2, 3, 3) from $\vec{BC} = \vec{AD}$ (1)
- $\vec{AB} \times \vec{AD} = -5\hat{i} - 4\hat{j} + 3\hat{k}$ (1)
- area $5\sqrt{2}$ square units (1)

11. Vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ satisfy $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$. Show that $\vec{a} - \vec{d}$ is parallel to $\vec{b} - \vec{c}$, given $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$. [3]

$(\vec{a} - \vec{d}) \times (\vec{b} - \vec{c}) = \vec{a} \times \vec{b} - \vec{a} \times \vec{c} - \vec{d} \times \vec{b} + \vec{d} \times \vec{c}$. Using $\vec{d} \times \vec{b} = -\vec{b} \times \vec{d}$ and $\vec{d} \times \vec{c} = -\vec{c} \times \vec{d}$, this is $\vec{a} \times \vec{b} - \vec{a} \times \vec{c} + \vec{b} \times \vec{d} - \vec{c} \times \vec{d}$. Substituting $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ gives $\vec{c} \times \vec{d} - \vec{b} \times \vec{d} + \vec{b} \times \vec{d} - \vec{c} \times \vec{d} = \vec{0}$. Both vectors are non-zero, so their vector product being $\vec{0}$ means they are parallel.

- expands $(\vec{a} - \vec{d}) \times (\vec{b} - \vec{c})$ by distributivity (1)
- uses anti-commutativity and the two given equalities to get $\vec{0}$ (1)
- concludes parallel since both are non-zero (1)