Class 12 Mathematics · Chapter 11: Three Dimensional Geometry

Answer key and marking scheme · 20 marks · about 40 minutes · based on NCERT

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1. The line joining the origin O to the point P(1, 1, $\sqrt{2}$) makes with the positive direction of the z-axis an angle of [1]

(a) 60°
(b) 45°
(c) 30°
(d) 90°

Answer: (b) OP = $\sqrt{(1 + 1 + 2)} = 2$, so $\cos\gamma = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$ and $\gamma = 45°$. Option (a) takes $\cos\gamma = \frac{1}{2}$ from the x or y coordinate.

2. The line $\vec{r} = 3\hat{i} - 2\hat{k} + \lambda\hat{j}$ [1]

(a) is perpendicular to the y-axis and passes through (3, 0, −2)
(b) is parallel to the y-axis and passes through the origin
(c) is parallel to the vector $3\hat{i} - 2\hat{k}$ and passes through (0, 1, 0)
(d) is parallel to the y-axis and passes through the point (3, 0, −2)

Answer: (d) In $\vec{r} = \vec{a} + \lambda\vec{b}$, $\vec{a} = 3\hat{i} - 2\hat{k}$ is the position vector of a point on the line, (3, 0, −2), and $\vec{b} = \hat{j}$ gives direction ratios 0, 1, 0, the direction of the y-axis. Option (c) swaps the roles of $\vec{a}$ and $\vec{b}$; (a) is wrong because the direction is that of the y-axis itself.

3. Two lines in space that are neither parallel nor intersecting are called [1]

(a) coplanar lines
(b) skew lines
(c) concurrent lines
(d) perpendicular lines

Answer: (b) Lines in space which are neither parallel nor intersecting are skew lines; they lie in different planes, so (a) is wrong. Perpendicular lines may intersect, and concurrent lines meet.

4. If 2k, −k and 2k are the direction cosines of a line, where k > 0, then k is [1]

(a) $\frac{1}{3}$
(b) $\frac{1}{9}$
(c) $\frac{1}{\sqrt{3}}$
(d) 3

Answer: (a) $(2k)^2 + (-k)^2 + (2k)^2 = 1$ gives $9k^2 = 1$, so $k = \frac{1}{3}$. Option (b) stops at k^2; (c) uses $3k^2 = 1$; (d) inverts.

5. A line has direction ratios 2, −3, 6. Which of the following is NOT a set of direction ratios of the same line? [1]

(a) −4, 6, −12
(b) 1, $-\frac{3}{2}$, 3
(c) −2, −3, −6
(d) 6, −9, 18

Answer: (c) Any set of direction ratios of the line is k(2, −3, 6) with $k \neq 0$: (a) is k = −2, (b) is $k = \frac{1}{2}$, (d) is k = 3. In (c) the first number needs k = −1 but the second needs k = 1, so (c) is not proportional to 2, −3, 6.

6. The points A(−1, 2, 1), B(1, a, 5) and C(5, 11, b) are collinear. Find a and b. [2]

Direction ratios of AB: 2, a − 2, 4; of AC: 6, 9, b − 1. For collinear points these are proportional: $\frac{6}{2} = 3$, so $9 = 3(a - 2)$ and $b - 1 = 3 \times 4$. Hence a = 5 and b = 13.

- direction ratios of AB and AC, set proportional (1)
- a = 5, b = 13 (1)

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7. For the line $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z-4}{-2}$, find (a) its direction cosines, (b) the point on it whose z-coordinate is 0. [2]

(a) Direction ratios 2, −1, −2; $\sqrt{(4+1+4)} = 3$, so the direction cosines are $\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}$ (or their negatives).

(b) A general point is $(1+2\lambda, -3-\lambda, 4-2\lambda)$. $4 - 2\lambda = 0$ gives $\lambda = 2$: the point is (5, −5, 0).

- direction cosines $\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}$ (1)
- $\lambda = 2$, point (5, −5, 0) (1)

8. Find the acute angle between the line $\frac{x-4}{2} = \frac{1-y}{2} = \frac{z}{1}$ and the y-axis. [2]

The line is $\frac{x-4}{2} = \frac{y-1}{-2} = \frac{z}{1}$, direction ratios 2, −2, 1. The y-axis has direction ratios 0, 1, 0. $\cos\theta = \frac{|0-2+0|}{3\times 1} = \frac{2}{3}$, so $\theta = \cos^{-1}(\frac{2}{3})$.

- direction ratios 2, −2, 1 (sign of y fixed) (1)
- $\theta = \cos^{-1}(\frac{2}{3})$ (1)

9. Find k for which the line through A(1, 0, −1) and B(2, 1, 1) meets the line through C(0, 2, 1) and D(1, 2, k), and find the point where they meet. [3]

l_1: $\vec{r} = \hat{i} - \hat{k} + \lambda(\hat{i} + \hat{j} + 2\hat{k})$; l_2: $\vec{r} = 2\hat{j} + \hat{k} + \mu(\hat{i} + (k-1)\hat{k})$. The direction ratios are not proportional, so the lines meet exactly when $(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = 0$. $\vec{b}_1 \times \vec{b}_2 = |\hat{i}\ \hat{j}\ \hat{k};\ 1\ 1\ 2;\ 1\ 0\ k-1| = (k-1)\hat{i} + (3-k)\hat{j} - \hat{k}$, and $\vec{a}_2 - \vec{a}_1 = -\hat{i} + 2\hat{j} + 2\hat{k}$, so the product is $-(k-1) + 2(3-k) - 2 = 5 - 3k = 0$, $k = 5/3$. Then $(1+\lambda, \lambda, -1+2\lambda) = (\mu, 2, 1+\frac{2}{3}\mu)$ gives $\lambda = 2$, $\mu = 3$, and z agrees (3 = 3). The lines meet at (3, 2, 3).

- condition $(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = 0$ giving $5 - 3k = 0$ (1)
- $k = \frac{5}{3}$ (1)
- $\lambda = 2$, $\mu = 3$, point (3, 2, 3) (1)

10. Show that the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{-1}$ and $\frac{x-2}{1} = \frac{y-4}{-2} = \frac{z+2}{3}$ intersect, and find their point of intersection. [3]

General points: $(1+2\lambda, -1+3\lambda, 2-\lambda)$ and $(2+\mu, 4-2\mu, -2+3\mu)$. Equating x and y: $2\lambda - \mu = 1$ and $3\lambda + 2\mu = 5$, so $\lambda = 1$, $\mu = 1$. Then z: $2 - 1 = 1$ and $-2 + 3 = 1$, equal, so the lines intersect. The point is (3, 2, 1).

- general points of both lines (1)
- $\lambda = 1$, $\mu = 1$ from two coordinates, third coordinate verified (1)
- point of intersection (3, 2, 1) (1)

11. The vertices of a triangle are A(3, 3, 1), B(1, 2, 3) and C(3, 2, 5). Using direction ratios, show that the triangle is right-angled at B, and find cos A. [3]

Direction ratios of BA: 2, 1, −2; of BC: 2, 0, 2. $2\times 2 + 1\times 0 + (-2)\times 2 = 0$, so BA $\perp$ BC and $\angle B = 90°$. At A: AB has direction ratios −2, −1, 2 (magnitude 3) and AC has 0, −1, 4 (magnitude $\sqrt{17}$). $\cos A = \frac{0+1+8}{3\sqrt{17}} = \frac{3}{\sqrt{17}}$.

- direction ratios of BA and BC (1)
- $a_1a_2 + b_1b_2 + c_1c_2 = 0$, so $\angle B = 90°$ (1)
- $\cos A = \frac{3}{\sqrt{17}}$ (1)