Class 12 Mathematics · Chapter 12: Linear Programming

Answer key and marking scheme · 20 marks · about 40 minutes · based on NCERT

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1. On a bounded feasible region, Z = ax + by takes its maximum value 48 at both corner points P(2, 8) and Q(6, 4). Which of the following is true? [1]

(a) Z takes the value 48 only at P and Q

(b) the maximum value of Z is 96, the sum of the values at P and Q

(c) Z also takes the value 48 at (4, 6)

(d) Z has no maximum value, since two corner points give it

Answer: (c) When two corner points produce the same maximum value, every point on the segment joining them gives the same maximum. (4, 6) is the midpoint of PQ, so Z = 48 there. (From 2a + 8b = 6a + 4b = 48, a = b = 4.8, and 4.8(4 + 6) = 48 confirms it.)

2. In a linear programming problem with decision variables x and y, which of the following is a non-negative restriction? [1]

(a) $x + y \le 12$

(b) $x - 2y \ge 0$

(c) $Z = 5x + 3y$

(d) $y \ge 0$

Answer: (d) The conditions $x \ge 0$, $y \ge 0$ are the non-negative restrictions. $x + y \le 12$ and $x - 2y \ge 0$ are other constraints ($x - 2y \ge 0$ allows negative values of y), and $Z = 5x + 3y$ is an objective function.

3. The corner points of a bounded feasible region are (0, 0), (6, 0), (4, 3) and (0, 5). The maximum value of Z = 2x + 5y on the region is [1]

(a) 23

(b) 12

(c) 25

(d) 37

Answer: (c) Z at the corners: (0, 0) → 0, (6, 0) → 12, (4, 3) → 8 + 15 = 23, (0, 5) → 25. The region is bounded, so the largest corner value, 25 at (0, 5), is the maximum. 37 = 2(6) + 5(5) combines coordinates of two different corners.

4. The feasible region of a system of linear inequalities is said to be bounded if [1]

(a) it can be enclosed within a circle

(b) it contains the origin

(c) it has at most four corner points

(d) it lies in the first quadrant

Answer: (a) By definition a feasible region is bounded if it can be enclosed within a circle; otherwise it is unbounded. A region in the first quadrant, or one containing the origin, can still extend indefinitely, and a region with only two corner points can be unbounded.

5. For $x \ge 0$, $y \ge 0$, the constraints $2x + 3y \le 6$ and $x + y \ge k$ have no feasible region exactly when [1]

(a) $k > 2$

(b) $k > 3$

(c) $k < 3$

(d) $k > 6$

Answer: (b) On the triangle $2x + 3y \le 6$, $x, y \ge 0$, with corners (0, 0), (3, 0) and (0, 2), the largest value of x + y is 3, at (3, 0). So some point satisfies $x + y \ge k$ exactly when $k \le 3$, and there is no feasible region when $k > 3$. $k > 2$ uses the corner (0, 2) only; $k > 6$ uses the right side of $2x + 3y \le 6$.

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6. The corner points of a bounded feasible region are A(2, 0), B(6, 0), C(5, 4) and D(1, 3). Find the maximum and minimum values of Z = 3x − 4y, and the points at which they occur. [2]

Z(A) = 6, Z(B) = 18, Z(C) = 15 − 16 = −1, Z(D) = 3 − 12 = −9. The region is bounded, so the maximum is 18 at B(6, 0) and the minimum is −9 at D(1, 3).

- Z at all four corners: 6, 18, −1, −9 (1)
- maximum 18 at (6, 0); minimum −9 at (1, 3) (1)

7. The corner points of a bounded feasible region are (0, 0), (6, 0), (4, 4) and (0, 5). The maximum of Z = px + 3y on the region occurs at both (6, 0) and (4, 4). Find p and the maximum value of Z. [2]

Equal values at the two corners: 6p = 4p + 12, so p = 6. Then Z = 36 at (6, 0) and at (4, 4), while Z(0, 5) = 15 and Z(0, 0) = 0, so the maximum value is 36 (attained at every point of the segment joining (6, 0) and (4, 4)).

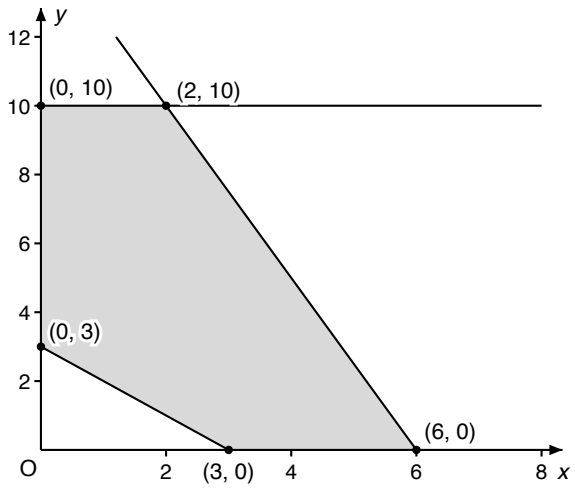
- equates 6p = 4p + 12, p = 6 (1)
- maximum 36, checked against the other corners (1)

8. The feasible region given by x + y ≥ 5, 3x + y ≥ 9, x ≥ 0, y ≥ 0 is unbounded, with corner points (0, 9), (2, 3) and (5, 0). The smallest value of Z = 4x + 3y at these corners is 17, at (2, 3). Show that 17 is the minimum value of Z on the region. [2]

Since the region is unbounded, check whether the open half plane 4x + 3y < 17 has a point in common with it. For any feasible point, 4x + 3y = (5/2)(x + y) + (1/2)(3x + y) ≥ (5/2)(5) + (1/2)(9) = 17. So no feasible point has 4x + 3y < 17; the half plane and the region have no common point, and 17 is the minimum, at (2, 3).

- states the test: the open half plane 4x + 3y < 17 must have no point in common with the region (1)
- shows 4x + 3y ≥ 17 on the region (or by the graph) and concludes (1)

9. The shaded region in the figure, with its boundary, is the feasible region of a linear programming problem. (a) Write the constraints that define it. (b) Find the maximum and minimum values of Z = 4x + y on it. [3]



(a) The boundary lines are x + y = 3 (through (3, 0), (0, 3)), 5x + 2y = 30 (through (6, 0), (2, 10)) and y = 10. The region lies on the side of x + y = 3 away from the origin and on the side of the other two containing it: x + y ≥ 3, 5x + 2y ≤ 30, y ≤ 10, x ≥ 0, y ≥ 0. (b) The region is bounded, with corners (3, 0), (6, 0), (2, 10), (0, 10), (0, 3); Z there: 12, 24, 18, 10, 3. Maximum 24 at (6, 0); minimum 3 at (0, 3).

- the three constraints with the correct inequality signs (1)
- Z at the five corners (1)
- maximum 24 at (6, 0), minimum 3 at (0, 3) (1)

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10. The feasible region of a linear programming problem is given by $x + y \le 7$, $2x - y \ge 0$, $x \le 4$, $x \ge 0$, $y \ge 0$. (a) Find all values of a for which the point $(a, 2)$ is a feasible solution. (b) Among these points, find the one at which $Z = 3x + y$ is largest, and that value of Z. [3]

(a) $(a, 2)$ must satisfy every constraint: $a + 2 \le 7$ gives $a \le 5$; $2a - 2 \ge 0$ gives $a \ge 1$; $a \le 4$; $a \ge 0$. Together $1 \le a \le 4$. (b) On these points $Z = 3a + 2$, which increases with a, so it is largest at $a = 4$: the point $(4, 2)$, $Z = 14$.

- each constraint turned into a condition on a (1)
- $1 \le a \le 4$ (1)
- $(4, 2)$, $Z = 14$ (1)

11. Kabir has at most 3 hours a day for exercise. He walks for x hours and cycles for y hours, and on his doctor's advice cycles for no longer than he walks. Walking burns 300 kcal an hour and cycling 500 kcal an hour. Formulate a linear programming problem to maximise the energy he burns, and solve it graphically. [3]

Maximise $Z = 300x + 500y$ subject to $x + y \le 3$, $y \le x$, $x \ge 0$, $y \ge 0$. The line $y = x$ passes through the origin; testing $(1, 0)$ shows $y \le x$ is the side below it. The bounded region has corners $(0, 0)$, $(3, 0)$ and $(3/2, 3/2)$, where $y = x$ meets $x + y = 3$. Z there: 0, 900, 1200. The maximum is 1200 kcal, by walking 1.5 hours and cycling 1.5 hours.

- formulation: $Z = 300x + 500y$; $x + y \le 3$, $y \le x$, $x, y \ge 0$ (1)
- corners $(0, 0)$, $(3, 0)$, $(3/2, 3/2)$ (1)
- maximum 1200 kcal at 1.5 h walking, 1.5 h cycling (1)

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