Class 12 Mathematics · Chapter 13: Probability

Answer key and marking scheme · 20 marks · about 40 minutes · based on NCERT

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1. In a town in Assam, the probability that it rains on a given morning is 0.3, and the probability that the school bus is late on a rainy morning is 0.6. The probability that on a given morning it rains and the bus is late is [1]

(a) 0.9 (b) 0.18
(c) 0.5 (d) 0.42

Answer: (b) With R the event 'it rains' and L 'the bus is late', $P(R \cap L) = P(R)\,P(L|R) = 0.3 \times 0.6 = 0.18$. Option (a) adds; (c) divides; (d) uses $P(R') = 0.7$.

2. A jar at a school fair holds n coupons, exactly 2 of which win a prize. Two coupons are drawn one after the other without replacement. If the probability that both coupons win is $\frac{1}{15}$, then n is [1]

(a) 5 (b) 15
(c) 6 (d) 30

Answer: (c) P(first wins) = 2/n and P(second wins | first wins) = 1/(n − 1), so $2/(n(n-1)) = 1/15$, $n(n-1) = 30$ and n = 6. Check: 2/6 × 1/5 = 1/15. Option (a) gives 1/10; (b) treats the draws as one draw of a winner.

3. In a lucky draw at a mela in Varanasi, one token is drawn at random from 40 tokens numbered 1 to 40, and E is the event 'the number is even'. The announcer gives one clue about the number. After which clue is the probability of E the greatest? [1]

(a) P(E | the number is prime) (b) P(E | the number is a multiple of 3)
(c) P(E | the number is a perfect square) (d) P(E | the number is less than 6)

Answer: (c) Each clue reduces the sample space. Primes up to 40: 12, of which only 2 is even, 1/12. Multiples of 3: 13, of which the 6 multiples of 6 are even, 6/13. Perfect squares {1, 4, 9, 16, 25, 36}: 3 even, 1/2. Less than 6: {1, 2, 3, 4, 5}, 2 even, 2/5. The greatest is 1/2, after the perfect-square clue.

4. A and B are independent events with P(A) = 0.3 and $P(A \cap B) = 0.12$. Then P(B′) is [1]

(a) 0.6 (b) 0.4
(c) 0.88 (d) 0.036

Answer: (a) For independent events $P(A \cap B) = P(A)\,P(B)$, so P(B) = 0.12/0.3 = 0.4 and P(B′) = 1 − 0.4 = 0.6. Option (b) is P(B); (c) is $1 - P(A \cap B)$.

5. For events E and F of a sample space, P(F) = 0.4 and P(E′|F) = 0.35. Then $P(E \cap F)$ is [1]

(a) 0.26 (b) 0.14
(c) 0.65 (d) 0.05

Answer: (a) P(E|F) = 1 − P(E′|F) = 0.65, so $P(E \cap F) = P(F)\,P(E|F) = 0.4 \times 0.65 = 0.26$. Option (b) uses 0.35 in place of 0.65; (c) stops at P(E|F); (d) subtracts.

6. For events A and B, P(A) = 0.5, P(B) = 0.4 and $P(A \cup B) = 0.7$. Find P(A′|B′). [2]

$P(A' \cap B') = P((A \cup B)') = 1 - 0.7 = 0.3$ and P(B′) = 1 − 0.4 = 0.6. So $P(A'|B') = P(A' \cap B')/P(B') = 0.3/0.6 = 0.5$.

- $P(A' \cap B') = 1 - P(A \cup B) = 0.3$ (1)
- P(B′) = 0.6 and P(A′|B′) = 0.5 (1)

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7. A and B are independent events with P(A) = 0.6 and P(A′ ∩ B) = 0.24. Find P(B) and P(A ∪ B). [2]

A′ and B are independent when A and B are, so P(A′ ∩ B) = P(A′) P(B) = 0.4 P(B) = 0.24 and P(B) = 0.6. Then P(A ∪ B) = P(A) + P(B) − P(A) P(B) = 0.6 + 0.6 − 0.36 = 0.84.

- uses P(A′ ∩ B) = P(A′) P(B) to get P(B) = 0.6 (1)
- P(A ∪ B) = 0.84 (1)

8. A number is chosen at random from {1, 2, 3, 4}. Let A = {1, 2}, B = {1, 3} and C = {1, 4}. Show that each pair of these events is independent, but A, B and C are not mutually independent. [2]

P(A) = P(B) = P(C) = 1/2. A ∩ B = A ∩ C = B ∩ C = {1}, each of probability 1/4 = 1/2 × 1/2, so each pair is independent. But A ∩ B ∩ C = {1} has probability 1/4, while P(A) P(B) P(C) = 1/8; one of the four conditions fails, so the three events are not mutually independent.

- each pairwise intersection is {1}, probability 1/4 = 1/2 × 1/2 (1)
- P(A ∩ B ∩ C) = 1/4 ≠ 1/8, so not mutually independent (1)

9. A survey of 200 commuters in Hyderabad gave the table shown, by mode of travel and arrival. A commuter is chosen at random. Find (i) P(on time | travels by metro), (ii) P(travels by metro | on time), and (iii) state, with reason, whether 'travels by metro' and 'on time' are independent. [3]

Mode	On time	Late
Metro	72	8
Bus	84	36

(i) Of 80 metro commuters, 72 are on time: 72/80 = 9/10. (ii) Of 156 commuters on time, 72 used the metro: 72/156 = 6/13. (iii) P(M) = 80/200 = 0.4, P(T) = 156/200 = 0.78, P(M ∩ T) = 72/200 = 0.36, while P(M) P(T) = 0.312 ≠ 0.36; so the events are not independent (equally, P(T|M) = 0.9 ≠ P(T) = 0.78).

- (i) 9/10 (1)
- (ii) 6/13 (1)
- (iii) compares P(M ∩ T) = 0.36 with P(M) P(T) = 0.312: dependent (1)

10. Meera's alarm rings on a school day with probability 0.9. If it rings, she catches the 7:10 bus with probability 0.8; if it does not, with probability 0.3. (i) Find the probability that she catches the bus. (ii) On a day she missed the bus, find the probability that her alarm had rung. [3]

(i) P(C) = 0.9 × 0.8 + 0.1 × 0.3 = 0.72 + 0.03 = 0.75. (ii) P(missed) = 0.25 and P(rang and missed) = 0.9 × 0.2 = 0.18, so P(rang | missed) = 0.18/0.25 = 0.72.

- (i) total probability 0.75 (1)
- (ii) P(rang ∩ missed) = 0.18 and P(missed) = 0.25 (1)
- 0.72 (1)

11. Of the calls to a mobile network's helpline, 60% are about billing, 25% about the network and 15% about other matters. A billing call is resolved in the first call with probability 0.9 and any other call with probability 0.5. The network team resolves a fraction r of its calls in the first call. Find r if a call chosen at random that was not resolved in the first call is equally likely to have been about billing or about the network. [3]

Let E_1, E_2 be 'billing' and 'network', and A 'not resolved in the first call'. $P(E_1 \cap A) = 0.6 \times 0.1 = 0.06$ and $P(E_2 \cap A) = 0.25(1 - r)$. By Bayes' theorem $P(E_1|A)$ and $P(E_2|A)$ have the same denominator P(A), so they are equal exactly when $P(E_1 \cap A) = P(E_2 \cap A)$: $0.25(1 - r) = 0.06$, $1 - r = 0.24$, $r = 0.76$.

- P(billing ∩ not resolved) = 0.06 (1)
- equal posteriors reduced to 0.25(1 − r) = 0.06 (1)
- r = 0.76 (1)

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