Class 12 Mathematics · Chapter 2: Inverse Trigonometric Functions

Answer key and marking scheme · 20 marks · about 40 minutes · based on NCERT

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1. Which of the following can never be the principal value of $\cos^{-1} x$, whatever $x \in [-1, 1]$ is? [1]

(a) $-\pi/6$ (b) 0
(c) π (d) $5\pi/6$

Answer: (a) Principal values of $\cos^{-1}$ lie in $[0, \pi]$. $0 = \cos^{-1} 1$, $\pi = \cos^{-1}(-1)$ and $5\pi/6 = \cos^{-1}(-\sqrt{3}/2)$ are attained; $-\pi/6$ lies outside $[0, \pi]$.

2. If $\cot^{-1} x = 5\pi/6$, then x equals [1]

(a) $-1/\sqrt{3}$ (b) $\sqrt{3}$
(c) $-\sqrt{3}$ (d) $1/\sqrt{3}$

Answer: (c) $5\pi/6$ lies in $(0, \pi)$, so $x = \cot(5\pi/6) = -\cot(\pi/6) = -\sqrt{3}$.

3. The domain of the function $g(x) = \sec^{-1}(2x + 1)$ is [1]

(a) $(-1, 0)$ (b) $[-1, 0]$
(c) $(-\infty, -1] \cup [0, \infty)$ (d) $(-\infty, -1) \cup (0, \infty)$

Answer: (c) $\sec^{-1}$ is defined on $R - (-1, 1)$: $2x + 1 \le -1$ or $2x + 1 \ge 1$, so $x \le -1$ or $x \ge 0$. The end points are included, since $\sec^{-1}(-1) = \pi$ and $\sec^{-1}(1) = 0$.

4. The domain of the function $f(x) = \sin^{-1}(3x - 2)$ is [1]

(a) $[-1, 1]$ (b) $[1/3, 1]$
(c) $[-1/3, 1]$ (d) $[1, 3]$

Answer: (b) $\sin^{-1}$ is defined on $[-1, 1]$: $-1 \le 3x - 2 \le 1$ gives $1 \le 3x \le 3$, that is $1/3 \le x \le 1$.

5. If $\cos^{-1} x = 5\pi/6$, then $\sin^{-1} x$ equals [1]

(a) $\pi/3$ (b) $-\pi/3$
(c) $-\pi/6$ (d) $2\pi/3$

Answer: (b) $x = \cos(5\pi/6) = -\sqrt{3}/2$. $\sin^{-1}(-\sqrt{3}/2)$ must lie in $[-\pi/2, \pi/2]$, and $\sin(-\pi/3) = -\sqrt{3}/2$, so $\sin^{-1} x = -\pi/3$.

6. Find the value of $\tan^{-1}(\tan 2) + \cos^{-1}(\cos 5)$, the angles being in radians. [2]

$\pi/2 < 2 < \pi$, and $\tan 2 = \tan(2 - \pi)$ with $2 - \pi \in (-\pi/2, \pi/2)$, so $\tan^{-1}(\tan 2) = 2 - \pi$. $3\pi/2 < 5 < 2\pi$, and $\cos 5 = \cos(2\pi - 5)$ with $2\pi - 5 \in [0, \pi]$, so $\cos^{-1}(\cos 5) = 2\pi - 5$. Sum $= (2 - \pi) + (2\pi - 5) = \pi - 3$.

- $\tan^{-1}(\tan 2) = 2 - \pi$, with the branch stated (1)
- $\cos^{-1}(\cos 5) = 2\pi - 5$, and sum $\pi - 3$ (1)

7. Write $\sin^{-1}(x/\sqrt{(1 + x^2)})$, $x \in R$, in the simplest form. [2]

Put $x = \tan\theta$ with $\theta = \tan^{-1} x \in (-\pi/2, \pi/2)$. Then $\sqrt{(1 + x^2)} = \sec\theta$ (positive, since $\cos\theta > 0$), so $x/\sqrt{(1 + x^2)} = \tan\theta \cos\theta = \sin\theta$. As $\theta \in [-\pi/2, \pi/2]$, $\sin^{-1}(\sin\theta) = \theta = \tan^{-1} x$.

- substitutes $x = \tan\theta$ and gets $\sin\theta$ (1)
- concludes $\tan^{-1} x$ with θ in the branch (1)

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8. Explain why the inverse of cosine is defined with its principal range $[0, \pi]$ and not $[-\pi/2, \pi/2]$. [2]

An inverse exists only on an interval where the function is one-one and onto. On $[-\pi/2, \pi/2]$ cosine is not one-one, since $\cos(-\pi/3) = \cos(\pi/3) = 1/2$, and it takes no negative values, so it is not onto $[-1, 1]$. On $[0, \pi]$ cosine falls steadily from 1 to -1, taking each value in $[-1, 1]$ exactly once, so it is one-one and onto $[-1, 1]$, and $\cos^{-1} : [-1, 1] \to [0, \pi]$ is defined.

- cos is not one-one (or not onto $[-1, 1]$) on $[-\pi/2, \pi/2]$, with an example (1)
- cos is one-one and onto $[-1, 1]$ on $[0, \pi]$ (1)

9. Let $f(x) = 2 \sin^{-1}(1 - x)$. (a) Find the domain and the range of f. (b) Find x for which $f(x) = \pi/3$. [3]

(a) $\sin^{-1}$ needs $-1 \le 1 - x \le 1$, that is $0 \le x \le 2$: domain $[0, 2]$. As $1 - x$ runs over $[-1, 1]$, $\sin^{-1}(1 - x)$ runs over $[-\pi/2, \pi/2]$, so f runs over $[-\pi, \pi]$: range $[-\pi, \pi]$. (b) $2 \sin^{-1}(1 - x) = \pi/3$ gives $\sin^{-1}(1 - x) = \pi/6$, which lies in $[-\pi/2, \pi/2]$, so $1 - x = \sin(\pi/6) = 1/2$ and $x = 1/2$, which is in the domain.

- domain $[0, 2]$ (1)
- range $[-\pi, \pi]$ (1)
- $x = 1/2$ (1)

10. Show that $\cos^{-1}(2x^2 - 1) = 2 \cos^{-1} x$ for $0 \le x \le 1$, and $\cos^{-1}(2x^2 - 1) = 2\pi - 2 \cos^{-1} x$ for $-1 \le x \le 0$. [3]

Put $x = \cos\theta$, $\theta = \cos^{-1} x \in [0, \pi]$. Then $2x^2 - 1 = 2 \cos^2\theta - 1 = \cos 2\theta$. If $0 \le x \le 1$, $\theta \in [0, \pi/2]$, so $2\theta \in [0, \pi]$ and $\cos^{-1}(\cos 2\theta) = 2\theta = 2 \cos^{-1} x$. If $-1 \le x \le 0$, $\theta \in [\pi/2, \pi]$, so $2\theta \in [\pi, 2\pi]$, outside the branch; but $\cos 2\theta = \cos(2\pi - 2\theta)$ with $2\pi - 2\theta \in [0, \pi]$, so $\cos^{-1}(\cos 2\theta) = 2\pi - 2\theta = 2\pi - 2 \cos^{-1} x$.

- $x = \cos\theta$ gives $\cos 2\theta$ (1)
- case $0 \le x \le 1$: 2θ in $[0, \pi]$, result $2 \cos^{-1} x$ (1)
- case $-1 \le x \le 0$: uses $2\pi - 2\theta$ in $[0, \pi]$ (1)

11. Find the value of $\sin(2 \tan^{-1}(1/3)) + \cos(\tan^{-1}(2\sqrt{2}))$. [3]

Let $\alpha = \tan^{-1}(1/3)$. $\sin 2\alpha = 2 \tan\alpha/(1 + \tan^2\alpha) = (2/3)/(10/9) = 3/5$. Let $\beta = \tan^{-1}(2\sqrt{2}) \in (0, \pi/2)$, so $\tan\beta = 2\sqrt{2}$, $\sec^2\beta = 1 + 8 = 9$ and $\cos\beta = 1/3$ (positive in $(0, \pi/2)$). Sum $= 3/5 + 1/3 = 14/15$.

- $\sin(2 \tan^{-1}(1/3)) = 3/5$ (1)
- $\cos(\tan^{-1} 2\sqrt{2}) = 1/3$ with the sign justified (1)
- sum 14/15 (1)