Class 12 Mathematics · Chapter 3: Matrices

Answer key and marking scheme · 20 marks · about 40 minutes · based on NCERT

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1. A matrix has 12 elements and more rows than columns. The number of different orders it can have is [1]

(a) 6 (b) 2
(c) 4 (d) 3

Answer: (d) An m × n matrix has mn elements, so mn = 12 with m > n. The pairs are 12 × 1, 6 × 2 and 4 × 3, so there are 3 orders.

2. If A = [[2, −1, 3], [0, 4, 1]] and B = [[1, 2], [5, −3], [−2, 0]], the element in the second row and first column of AB is [1]

(a) −9 (b) −12
(c) 18 (d) 22

Answer: (c) The (2, 1) element of AB takes row 2 of A and column 1 of B: (0)(1) + (4)(5) + (1)(−2) = 0 + 20 − 2 = 18.

3. A = $[a_{ij}]$ is a 2 × 3 matrix with $a_{ij} = ij - 2$. The sum of the elements in the second row of A is [1]

(a) 12 (b) 2
(c) 0 (d) 6

Answer: (d) $a_{21} = 2 - 2 = 0$, $a_{22} = 4 - 2 = 2$, $a_{23} = 6 - 2 = 4$, so A = [[−1, 0, 1], [0, 2, 4]] and the second row sums to 0 + 2 + 4 = 6.

4. The matrix [[2x − 1, y − 3], [0, 5]] is a scalar matrix. Then x + y equals [1]

(a) 6 (b) 3
(c) 9 (d) 0

Answer: (a) A scalar matrix is diagonal, so y − 3 = 0, y = 3; its diagonal elements are equal, so 2x − 1 = 5, x = 3. Hence x + y = 6.

5. Which of the following statements is true? [1]

(a) Every scalar matrix is an identity matrix. (b) Every diagonal matrix is a scalar matrix.
(c) Every zero matrix is a square matrix. (d) Every identity matrix is a scalar matrix.

Answer: (d) An identity matrix is a diagonal matrix whose diagonal elements are all equal to 1, so it is a scalar matrix with k = 1. A scalar matrix with k = 3 is not an identity matrix; diag(1, 2) is diagonal but not scalar; [0, 0] is a zero matrix of order 1 × 2, not square.

6. Find p, q, r and s if [[p + q, q − r], [2r + s, p − s]] = [[6, 1], [9, 2]]. [2]

Equating corresponding elements: p + q = 6, q − r = 1, 2r + s = 9, p − s = 2. Write p = s + 2, q = 6 − p = 4 − s, r = q − 1 = 3 − s. Then 2(3 − s) + s = 9 gives 6 − s = 9, s = −3. Hence p = −1, q = 7, r = 6, s = −3.

- four equations from equality of matrices (1)
- p = −1, q = 7, r = 6, s = −3 (1)

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7. Find x, y and z if the matrix $[[0, 2x - 1, 4], [3, 0, y + 2], [z, -5, 0]]$ is skew symmetric. [2]

For a skew symmetric matrix $a_{ji} = -a_{ij}$. From $a_{21} = 3$: $a_{12} = 2x - 1 = -3$, so $x = -1$. From $a_{32} = -5$: $a_{23} = y + 2 = 5$, so $y = 3$. From $a_{13} = 4$: $a_{31} = z = -4$. The diagonal elements are already 0.

- uses $a_{ji} = -a_{ij}$ (1)
- $x = -1$, $y = 3$, $z = -4$ (1)

8. Construct the 2×3 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = |2i - 3j|$. [2]

$a_{11} = |2 - 3| = 1$, $a_{12} = |2 - 6| = 4$, $a_{13} = |2 - 9| = 7$; $a_{21} = |4 - 3| = 1$, $a_{22} = |4 - 6| = 2$, $a_{23} = |4 - 9| = 5$. So $A = [[1, 4, 7], [1, 2, 5]]$.

- first row 1, 4, 7 (1)
- second row 1, 2, 5, written as a 2×3 matrix (1)

9. Express $C = [[1, 4, -2], [0, 3, 5], [6, -1, 2]]$ as the sum of a symmetric and a skew symmetric matrix. [3]

$C' = [[1, 0, 6], [4, 3, -1], [-2, 5, 2]]$. $P = (1/2)(C + C') = (1/2)[[2, 4, 4], [4, 6, 4], [4, 4, 4]] = [[1, 2, 2], [2, 3, 2], [2, 2, 2]]$, and $P' = P$. $Q = (1/2)(C - C') = (1/2)[[0, 4, -8], [-4, 0, 6], [8, -6, 0]] = [[0, 2, -4], [-2, 0, 3], [4, -3, 0]]$, and $Q' = -Q$. $P + Q = C$.

- C' and $P = (1/2)(C + C')$, symmetric (1)
- $Q = (1/2)(C - C')$, skew symmetric (1)
- $P + Q = C$ stated (1)

10. For a real number t, let $G(t) = [[1, t], [0, 1]]$. (a) Show that $G(s)G(t) = G(s + t)$. (b) Hence show that $G(t)$ is invertible with $G(t)^{-1} = G(-t)$, and find $[G(2)]^3$. [3]

(a) $G(s)G(t) = [[1\cdot1 + s\cdot0, 1\cdot t + s\cdot1], [0, 1]] = [[1, s + t], [0, 1]] = G(s + t)$. (b) $G(t)G(-t) = G(0) = I$ and $G(-t)G(t) = G(0) = I$, so $G(-t)$ is the inverse of $G(t)$. $[G(2)]^3 = G(2)G(2)G(2) = G(4)G(2) = G(6) = [[1, 6], [0, 1]]$.

- product $G(s)G(t) = G(s + t)$ (1)
- $G(t)G(-t) = G(-t)G(t) = I$, so $G(t)^{-1} = G(-t)$ (1)
- $[G(2)]^3 = [[1, 6], [0, 1]]$ (1)

11. Find matrices X and Y if $2X + Y = [[5, 0], [-3, 10]]$ and $X - 2Y = [[0, 5], [-4, 0]]$. [3]

Let $A = [[5, 0], [-3, 10]]$, $B = [[0, 5], [-4, 0]]$. Multiplying the first equation by 2 and adding the second: $5X = 2A + B = [[10, 5], [-10, 20]]$, so $X = [[2, 1], [-2, 4]]$. Then $Y = A - 2X = [[5, 0], [-3, 10]] - [[4, 2], [-4, 8]] = [[1, -2], [1, 2]]$. Check: $X - 2Y = [[0, 5], [-4, 0]]$.

- eliminates Y: $5X = 2A + B$ (1)
- $X = [[2, 1], [-2, 4]]$ (1)
- $Y = [[1, -2], [1, 2]]$ (1)

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