Class 12 Mathematics · Chapter 4: Determinants

Answer key and marking scheme · 20 marks · about 40 minutes · based on NCERT

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1. The value of |A| for A = [[3, 0, 2], [1, 4, 0], [−2, 1, 5]] is [1]

(a) 60 (b) 42
(c) 78 (d) −78

Answer: (c) Expand along R_1 (it has a zero): |A| = 3(4·5 − 0·1) − 0 + 2(1·1 − 4·(−2)) = 3(20) + 2(9) = 60 + 18 = 78.

2. If |[[x, 4], [3, x]]| = |[[5, 2], [7, 4]]| and x > 0, then x is [1]

(a) $3\sqrt{2}$ (b) $2\sqrt{3}$
(c) 3 (d) $\sqrt{6}$

Answer: (a) $x^2 - 12 = 20 - 14 = 6$, so $x^2 = 18$ and, as x > 0, $x = \sqrt{18} = 3\sqrt{2}$.

3. The system x + 2y = 3, 2x + 4y = 7 is [1]

(a) inconsistent, as it has no solution (b) consistent, with exactly two solutions
(c) consistent, with infinitely many solutions (d) consistent, with a unique solution

Answer: (a) Doubling the first equation gives 2x + 4y = 6, which cannot also equal 7. No (x, y) satisfies both, so the system is inconsistent.

4. In the determinant |[[2, 7, −3], [1, 0, 4], [5, −2, 6]]|, the cofactor A_{23} of the element 4 is [1]

(a) −31 (b) 31
(c) −39 (d) 39

Answer: (d) M_{23} = |[[2, 7], [5, −2]]| = −4 − 35 = −39, and $A_{23} = (-1)^{2+3} M_{23} = -(-39) = 39$.

5. The matrix [[k, 6], [3, 2]] is singular when k is [1]

(a) −9 (b) 4
(c) 9 (d) 1

Answer: (c) Singular means the determinant is 0: 2k − 18 = 0, so k = 9.

6. Using determinants, find the equation of the line joining the points (2, 3) and (−1, −3). [2]

For P(x, y) on the line, the area of the triangle with vertices (x, y), (2, 3), (−1, −3) is zero: |[[x, y, 1], [2, 3, 1], [−1, −3, 1]]| = 0. Expanding along R_1: x(3 + 3) − y(2 + 1) + 1(−6 + 3) = 6x − 3y − 3 = 0, i.e. 2x − y − 1 = 0. (Both points satisfy it.)

- sets the determinant with (x, y) and the two points equal to zero (1)
- expands to 2x − y − 1 = 0 (1)

7. For Δ = |[[2, −1, 4], [0, 3, 1], [5, 2, −2]]|, find the cofactors A_{11}, A_{12}, A_{13} and hence the value of Δ. [2]

A_{11} = |[[3, 1], [2, −2]]| = −6 − 2 = −8; A_{12} = −|[[0, 1], [5, −2]]| = −(0 − 5) = 5; A_{13} = |[[0, 3], [5, 2]]| = 0 − 15 = −15. $\Delta = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = 2(-8) + (-1)(5) + 4(-15) = -16 - 5 - 60 = -81$.

- the three cofactors −8, 5, −15 (1)
- Δ = −81 (1)

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8. Find the values of λ for which the matrix $[[\lambda, 2, 0], [2, \lambda, 0], [1, 1, 3]]$ is singular. [2]

Expand along C_3, which has two zeros: the determinant is $3\,|[[\lambda, 2], [2, \lambda]]| = 3(\lambda^2 - 4)$. It is singular when $3(\lambda^2 - 4) = 0$, i.e. $\lambda = 2$ or $\lambda = -2$.

- determinant $3(\lambda^2 - 4)$, expanded along C_3 (1)
- $\lambda = \pm 2$ (1)

9. For $\Delta = |[[2, 0, 3], [-1, 4, 1], [5, 0, -2]]|$, (a) evaluate Δ using the cofactors of the elements of the second column, and (b) verify that $a_{11}A_{12} + a_{21}A_{22} + a_{31}A_{32} = 0$. [3]

(a) $A_{12} = -|[[-1, 1], [5, -2]]| = -(2 - 5) = 3$; $A_{22} = |[[2, 3], [5, -2]]| = -4 - 15 = -19$; $A_{32} = -|[[2, 3], [-1, 1]]| = -(2 + 3) = -5$. $\Delta = 0(3) + 4(-19) + 0(-5) = -76$. (b) $a_{11}A_{12} + a_{21}A_{22} + a_{31}A_{32} = 2(3) + (-1)(-19) + 5(-5) = 6 + 19 - 25 = 0$: the elements of C_1 times the cofactors of C_2.

- the cofactors 3, −19, −5 (1)
- $\Delta = -76$ (1)
- (b) sum $6 + 19 - 25 = 0$ (1)

10. Show that $A = [[0, 1, 1], [1, 0, 1], [1, 1, 0]]$ satisfies $A^2 - A - 2I = O$, and hence find A^{-1}. [3]

$A^2 = [[2, 1, 1], [1, 2, 1], [1, 1, 2]]$, so $A^2 - A - 2I = [[2 - 0 - 2, 1 - 1, 1 - 1], [1 - 1, 2 - 0 - 2, 1 - 1], [1 - 1, 1 - 1, 2 - 0 - 2]] = O$. $|A| = 0 - 1(0 - 1) + 1(1 - 0) = 2 \neq 0$, so A^{-1} exists. Post-multiplying $A^2 - A = 2I$ by A^{-1}: $A - I = 2A^{-1}$, so $A^{-1} = \frac{1}{2}(A - I) = \frac{1}{2}[[-1, 1, 1], [1, -1, 1], [1, 1, -1]]$.

- A^2 computed and the identity shown (1)
- $|A| \neq 0$ and $A - I = 2A^{-1}$ (1)
- $A^{-1} = \frac{1}{2}[[-1, 1, 1], [1, -1, 1], [1, 1, -1]]$ (1)

11. A sports teacher in Pune pays ₹5,600 for 3 footballs and 4 cricket bats, and ₹4,900 for 5 footballs and 2 cricket bats. Using the matrix method, find the price of a football and of a bat. [3]

Let x, y be the prices (₹) of a football and a bat: $3x + 4y = 5600$, $5x + 2y = 4900$, i.e. $AX = B$ with $A = [[3, 4], [5, 2]]$, $B = [[5600], [4900]]$. $|A| = 6 - 20 = -14 \neq 0$. adj $A = [[2, -4], [-5, 3]]$. $X = A^{-1}B = -\frac{1}{14}[[2(5600) - 4(4900)], [-5(5600) + 3(4900)]] = -\frac{1}{14}[[-8400], [-13300]] = [[600], [950]]$. A football costs ₹600 and a bat ₹950.

- equations as $AX = B$ and $|A| = -14$ (1)
- adj A and A^{-1} (1)
- $x = 600$, $y = 950$ (1)

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