Class 12 Mathematics · Chapter 5: Continuity and Differentiability

Answer key and marking scheme · 20 marks · about 40 minutes · based on NCERT

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1. Let f(x) = 2x + 1 for x < 3, f(3) = 4, and f(x) = x + 4 for x > 3. At x = 3, f is [1]

(a) continuous, since the left and right hand limits are equal

(b) discontinuous, since the left and right hand limits are unequal

(c) not defined, so the question of continuity does not arise

(d) discontinuous, since the limit at 3 exists but is not equal to f(3)

Answer: (d) Left hand limit = 2(3) + 1 = 7; right hand limit = 3 + 4 = 7; so $\lim_{x\to 3} f(x) = 7$, but f(3) = 4 ≠ 7. The limit exists and differs from the value, so f is discontinuous at 3.

2. $\frac{d}{dx}\cos(x^3)$ is [1]

(a) $-3x^2 \sin(x^3)$

(b) $-\sin(x^3)$

(c) $3x^2 \sin(x^3)$

(d) $-3x^2 \cos(x^3)$

Answer: (a) With $t = x^3$: $d/dx \cos t = -\sin t \cdot dt/dx = -\sin(x^3) \cdot 3x^2$.

3. The derivative of 5^x at x = 0 is [1]

(a) 0

(b) 5 log 5

(c) 1

(d) log 5

Answer: (d) $d/dx\,(a^x) = a^x \log a$, so the derivative is $5^x \log 5$, which at x = 0 is $5^0 \log 5 = \log 5$.

4. The function $f(x) = \frac{x^3 - 8}{x - 2}$ for x ≠ 2, and f(2) = k, is continuous at x = 2. The value of k is [1]

(a) 4

(b) 6

(c) 12

(d) 0

Answer: (c) For x ≠ 2, $(x^3 - 8)/(x - 2) = x^2 + 2x + 4$, so $\lim_{x\to 2} f(x) = 4 + 4 + 4 = 12$. Continuity at 2 needs f(2) = 12, so k = 12.

5. Which statement about $f(x) = \frac{1}{x}$, x ≠ 0, is correct? [1]

(a) f is a continuous function, since it is continuous at every point of its domain

(b) f is discontinuous at x = 0, so it is not a continuous function

(c) f is continuous only for x > 0

(d) $\lim_{x\to 0^+} f(x)$ exists and equals +∞, a real number

Answer: (a) 0 is not in the domain, so continuity at 0 is not asked. At every c ≠ 0, $\lim_{x\to c} 1/x = 1/c = f(c)$, so f is a continuous function. +∞ is not a real number, so (d) is false.

6. If $x = e^t + e^{-t}$ and $y = e^t - e^{-t}$, show that $\frac{dy}{dx} = \frac{x}{y}$. [2]

$dx/dt = e^t - e^{-t} = y$ and $dy/dt = e^t + e^{-t} = x$, so $dy/dx = (dy/dt)/(dx/dt) = x/y$ (for t ≠ 0, where y ≠ 0).

- dx/dt = y and dy/dt = x (1)
- dy/dx = x/y (1)

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7. If $y = \log(x^2 + 1)$, find $\frac{d^2y}{dx^2}$. [2]

$dy/dx = 2x/(x^2 + 1)$. By the quotient rule, $d^2y/dx^2 = [2(x^2 + 1) - 2x \cdot 2x]/(x^2 + 1)^2 = 2(1 - x^2)/(x^2 + 1)^2$.

- $dy/dx = 2x/(x^2 + 1)$ (1)
- $d^2y/dx^2 = 2(1 - x^2)/(x^2 + 1)^2$ (1)

8. Find k if $f(x) = kx + 3$ for $x \le 2$ and $f(x) = x^2 - k$ for $x > 2$ is continuous at $x = 2$. [2]

Left hand limit and f(2): $2k + 3$. Right hand limit: $4 - k$. Continuity needs $2k + 3 = 4 - k$, so $3k = 1$ and $k = 1/3$.

- writes $2k + 3 = 4 - k$ from the two sides (1)
- $k = 1/3$ (1)

9. Show that $f(x) = x^2$ for $x \le 1$ and $f(x) = 2x - 1$ for $x > 1$ is differentiable at $x = 1$. [3]

Continuity: $f(1) = 1$, left hand limit 1, right hand limit $2 - 1 = 1$. Left hand derivative: $\lim_{h \to 0^-} [(1 + h)^2 - 1]/h = \lim (2 + h) = 2$. Right hand derivative: $\lim_{h \to 0^+} [2(1 + h) - 1 - 1]/h = 2$. They are finite and equal, so f is differentiable at 1 with $f'(1) = 2$.

- continuity at $x = 1$ checked (1)
- left hand derivative = 2 (1)
- right hand derivative = 2, so differentiable (1)

10. Find a and b so that $f(x) = ax + 5$ for $x \le 1$, $f(x) = x^2 + b$ for $1 < x < 3$, and $f(x) = 2bx - 1$ for $x \ge 3$ is a continuous function. [3]

Each piece is a polynomial, so f is continuous except possibly at the joins. At $x = 1$: $f(1) = a + 5$ and the right hand limit is $1 + b$, so $a + 5 = 1 + b$. At $x = 3$: the left hand limit is $9 + b$ and $f(3) = 6b - 1$, so $9 + b = 6b - 1$, giving $b = 2$. Then $a = 1 + 2 - 5 = -2$.

- condition at $x = 1$: $a + 5 = 1 + b$ (1)
- condition at $x = 3$: $9 + b = 6b - 1$, $b = 2$ (1)
- $a = -2$ (1)

11. For $x < 1$, let $y = \tan^{-1}\left(\frac{1 + x}{1 - x}\right)$. Find $\frac{dy}{dx}$ (i) by writing $x = \tan\theta$, and (ii) directly by the chain rule, and show that the two results agree. [3]

(i) With $x = \tan\theta$, $\theta \in (-\pi/2, \pi/4)$: $(1 + \tan\theta)/(1 - \tan\theta) = \tan(\pi/4 + \theta)$, and $\pi/4 + \theta \in (-\pi/4, \pi/2)$, so $y = \pi/4 + \theta = \pi/4 + \tan^{-1} x$ and $dy/dx = 1/(1 + x^2)$. (ii) Directly, $dy/dx = [1/(1 + u^2)]\, du/dx$ with $u = (1 + x)/(1 - x)$, $du/dx = 2/(1 - x)^2$; $1 + u^2 = [(1 - x)^2 + (1 + x)^2]/(1 - x)^2 = 2(1 + x^2)/(1 - x)^2$. So $dy/dx = 2/[2(1 + x^2)] = 1/(1 + x^2)$, the same.

- substitution gives $y = \pi/4 + \tan^{-1} x$, $dy/dx = 1/(1 + x^2)$ (1)
- chain rule with $du/dx = 2/(1 - x)^2$ (1)
- simplifies to $1/(1 + x^2)$: agrees (1)