Class 12 Mathematics · Chapter 6: Application of Derivatives

Answer key and marking scheme · 20 marks · about 40 minutes · based on NCERT

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1. Which of the following functions is increasing on R? [1]

(a) e^{-x} (b) $x^3 + 5x$

(c) x^2 (d) cos x

Answer: (b) For $x^3 + 5x$, $f'(x) = 3x^2 + 5 > 0$ for every real x. e^{-x} has derivative $-e^{-x} < 0$; x^2 decreases for $x < 0$; cos x changes direction.

2. The function $f(x) = 5 - 2x^3$ on R is [1]

(a) strictly increasing (b) strictly decreasing

(c) constant (d) neither increasing nor decreasing

Answer: (b) If $x_1 < x_2$, then $x_1^3 < x_2^3$, so $-2x_1^3 > -2x_2^3$ and $f(x_1) > f(x_2)$: f is strictly decreasing on R. $f'(x) = -6x^2$ is 0 at $x = 0$ only, which does not spoil strictness.

3. If f is continuous on the closed interval [a, b], then f [1]

(a) attains its absolute maximum and its absolute minimum value, each at least once, in [a, b] (b) attains its absolute maximum only at a or at b

(c) has a point c in (a, b) with $f'(c) = 0$ (d) has no local maxima in (a, b)

Answer: (a) Theorem 5: a continuous function on a closed interval attains its absolute maximum and absolute minimum, each at least once. f need not be differentiable, so (c) need not hold, and the maximum may be at an interior point.

4. The side of a square sheet of ice is decreasing at 0.5 cm/s as it melts. When the side is 8 cm, the rate of change of its area is [1]

(a) -16 cm^2/s (b) 8 cm^2/s

(c) -4 cm^2/s (d) -8 cm^2/s

Answer: (d) $A = x^2$, $dA/dt = 2x\, dx/dt = 2(8)(-0.5) = -8$ cm^2/s; the negative sign says the area is decreasing.

5. A cylindrical can has a fixed height of 5 cm. The rate of change of its volume with respect to its radius r, when r = 2 cm, is [1]

(a) 40π cm^3/cm (b) 10π cm^3/cm

(c) 20π cm^3/cm (d) 4π cm^3/cm

Answer: (c) $V = \pi r^2(5) = 5\pi r^2$, so $dV/dr = 10\pi r = 10\pi(2) = 20\pi$ cm^3 per cm of radius.

6. Find all the critical points of $f(x) = |x^2 - 4|$ on the interval $(-3, 3)$. [2]

For $|x| > 2$, $f(x) = x^2 - 4$ and $f'(x) = 2x$; for $|x| < 2$, $f(x) = 4 - x^2$ and $f'(x) = -2x$, which is 0 at $x = 0$. At $x = 2$ the left derivative is -4 and the right derivative 4; at $x = -2$ they are likewise -4 and 4. So f is not differentiable at $x = \pm 2$. Critical points: $x = 0$, $x = -2$, $x = 2$.

- $x = 0$, where $f'(x) = -2x = 0$ (1)
- $x = \pm 2$, where f is not differentiable (left and right derivatives differ) (1)

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7. Find the absolute maximum and absolute minimum values of $f(x) = x^3 - 6x^2 + 9x$ on $[0, 4]$, and the points at which each is attained. [2]

$f'(x) = 3x^2 - 12x + 9 = 3(x - 1)(x - 3) = 0$ at $x = 1, 3$. $f(0) = 0$, $f(1) = 4$, $f(3) = 0$, $f(4) = 4$. Absolute maximum 4, at $x = 1$ and $x = 4$; absolute minimum 0, at $x = 0$ and $x = 3$.

- critical points $x = 1, 3$ and values at 0, 1, 3, 4 (1)
- absolute maximum 4 ($x = 1, 4$), absolute minimum 0 ($x = 0, 3$) (1)

8. Using the second derivative test, decide whether $x = 2$ is a point of local maxima or local minima of $f(x) = x^3 - 12x + 1$, and find the value of f there. [2]

$f'(x) = 3x^2 - 12$, $f'(2) = 0$. $f''(x) = 6x$, $f''(2) = 12 > 0$, so $x = 2$ is a point of local minima, and the local minimum value is $f(2) = 8 - 24 + 1 = -15$.

- $f'(2) = 0$ and $f''(2) = 12 > 0$, so local minima (1)
- local minimum value $f(2) = -15$ (1)

9. Using the first derivative test, find the points of local maxima and local minima of $f(x) = x^3 - 3x^2 - 24x + 5$, and the local maximum and local minimum values. [3]

$f'(x) = 3x^2 - 6x - 24 = 3(x - 4)(x + 2)$, zero at $x = -2$ and 4. Through $x = -2$, f' changes from positive to negative: local maxima, value $f(-2) = -8 - 12 + 48 + 5 = 33$. Through $x = 4$, f' changes from negative to positive: local minima, value $f(4) = 64 - 48 - 96 + 5 = -75$.

- $f'(x) = 3(x - 4)(x + 2)$, critical points -2 and 4 (1)
- sign changes: local maxima at -2, local minima at 4 (1)
- local maximum value 33, local minimum value -75 (1)

10. Find the intervals in which $f(x) = x - 2 \sin x$, $0 \le x \le 2\pi$, is increasing and in which it is decreasing. [3]

$f'(x) = 1 - 2 \cos x = 0$ when $\cos x = 1/2$, that is $x = \pi/3, 5\pi/3$. On $[0, \pi/3)$, $\cos x > 1/2$, so $f' < 0$; on $(\pi/3, 5\pi/3)$, $\cos x < 1/2$, so $f' > 0$; on $(5\pi/3, 2\pi]$, $f' < 0$. So f is decreasing in $[0, \pi/3)$ and $(5\pi/3, 2\pi]$, and increasing in $(\pi/3, 5\pi/3)$.

- $f'(x) = 1 - 2 \cos x$, zero at $\pi/3$ and $5\pi/3$ (1)
- sign of f' on the three intervals (1)
- increasing in $(\pi/3, 5\pi/3)$; decreasing in $[0, \pi/3)$ and $(5\pi/3, 2\pi]$ (1)

11. A farmer has to fence a rectangular plot of area 800 m^2 that lies along a straight canal; the side along the canal needs no fence. Find the dimensions that use the least fencing, and that length. [3]

Let each side perpendicular to the canal be w m; the side along the canal is $800/w$ m. Fencing $L(w) = 2w + 800/w$. $L'(w) = 2 - 800/w^2 = 0$ at $w = 20$ ($w > 0$). $L''(w) = 1600/w^3 > 0$, a minimum. Dimensions 20 m by 40 m (40 m along the canal); least fencing $40 + 40 = 80$ m.

- $L(w) = 2w + 800/w$ (1)
- $L'(w) = 0$ at $w = 20$ with $L'' > 0$ (1)
- 20 m × 40 m, fencing 80 m (1)

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