Class 12 Mathematics · Chapter 7: Integrals

Answer key and marking scheme · 20 marks · about 40 minutes · based on NCERT

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1. $\int (\cos 4x - e^{-x})\,dx$ equals [1]

(a) $\frac{1}{4}\sin 4x - e^{-x} + C$ (b) $\frac{1}{4}\sin 4x + e^{-x} + C$

(c) $-\frac{1}{4}\sin 4x + e^{-x} + C$ (d) $4\sin 4x + e^{-x} + C$

Answer: (b) $\frac{d}{dx}(\frac{1}{4}\sin 4x) = \cos 4x$ and $\frac{d}{dx}(e^{-x}) = -e^{-x}$, so the integral is $\frac{1}{4}\sin 4x + e^{-x} + C$.

2. $\int \frac{dx}{x^2 + 25}$ equals [1]

(a) $\frac{1}{25}\tan^{-1}\frac{x}{5} + C$ (b) $\frac{1}{5}\tan^{-1}\frac{x}{5} + C$

(c) $\tan^{-1}\frac{x}{5} + C$ (d) $\frac{1}{10}\log\left|\frac{x-5}{x+5}\right| + C$

Answer: (b) $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$ with $a = 5$. Option (d) is the form for $x^2 - 25$.

3. $\int \operatorname{cosec} x\,dx$ equals [1]

(a) $-\operatorname{cosec} x \cot x + C$ (b) $\log|\operatorname{cosec} x + \cot x| + C$

(c) $\log|\operatorname{cosec} x - \cot x| + C$ (d) $\log|\sin x| + C$

Answer: (c) Multiplying and dividing by $(\operatorname{cosec} x - \cot x)$ and putting $\operatorname{cosec} x - \cot x = t$ gives $\int \operatorname{cosec} x\,dx = \log|\operatorname{cosec} x - \cot x| + C$. Option (d) is $\int \cot x\,dx$; (a) is the derivative of $\operatorname{cosec} x$.

4. F is an anti derivative of $f(x) = 6x^2 - 4x + 1$ with $F(1) = 5$. Then $F(2)$ equals [1]

(a) 18 (b) 10

(c) 14 (d) 9

Answer: (c) $F(x) = 2x^3 - 2x^2 + x + C$. $F(1) = 1 + C = 5$ gives $C = 4$, so $F(2) = 16 - 8 + 2 + 4 = 14$.

5. $\int (\sec x \tan x + 2x)\,dx$ equals [1]

(a) $\tan x + x^2 + C$ (b) $\sec x + x^2 + C$

(c) $\frac{1}{2}\sec^2 x + x^2 + C$ (d) $\sec x \tan x + x^2 + C$

Answer: (b) $d/dx\,(\sec x) = \sec x \tan x$ and $d/dx\,(x^2) = 2x$, so $\int (\sec x \tan x + 2x)\,dx = \sec x + x^2 + C$.

6. The slope of the tangent to a curve at (x, y) is $3x^2 - \frac{2}{x^2}$, and the curve passes through (1, 4). Find the equation of the curve. [2]

$y = \int (3x^2 - 2x^{-2})\,dx = x^3 + \frac{2}{x} + C$. At (1, 4): $4 = 1 + 2 + C$, so $C = 1$ and $y = x^3 + \frac{2}{x} + 1$.

- $y = x^3 + \frac{2}{x} + C$ (1)
- uses (1, 4) to get $C = 1$ and states the curve (1)

7. Find $\int \frac{e^x + 1}{e^x + x}\,dx$. [2]

Put $e^x + x = t$; then $(e^x + 1)\,dx = dt$ and the integral is $\int \frac{dt}{t} = \log|e^x + x| + C$.

- substitution $e^x + x = t$ with $dt = (e^x + 1)\,dx$ (1)
- $\log|e^x + x| + C$ (1)

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8. Evaluate $\int_0^1 \frac{x}{\sqrt{(1+3x^2)}}\,dx$. [2]

Put $t = 1 + 3x^2$, $dt = 6x\,dx$; $x = 0$ gives $t = 1$ and $x = 1$ gives $t = 4$. The integral is $\frac{1}{6}\int_1^4 t^{-1/2}\,dt = \frac{1}{6}[2\sqrt{t}]_1^4 = \frac{1}{6}(4 - 2) = \frac{1}{3}$.

- substitution with the limits changed to 1 and 4 (1)
- value $\frac{1}{3}$ (1)

9. Find $\int \sin^2 x \cos^2 x\,dx$. [3]

$\sin^2 x \cos^2 x = \frac{1}{4}\sin^2 2x = \frac{1}{8}(1 - \cos 4x)$. Hence $\int \sin^2 x \cos^2 x\,dx = \frac{x}{8} - \frac{\sin 4x}{32} + C$.

- writes the integrand as $\frac{1}{4}\sin^2 2x$ (1)
- uses $\sin^2\theta = \frac{1 - \cos 2\theta}{2}$ to get $\frac{1}{8}(1 - \cos 4x)$ (1)
- $\frac{x}{8} - \frac{\sin 4x}{32} + C$ (1)

10. Show that $\int_1^2 \frac{e^x}{e^x + e^{3-x}}\,dx = \frac{1}{2}$. [3]

Let I be the integral. By P_3 with $a + b = 3$, replacing x by $3 - x$: $I = \int_1^2 \frac{e^{3-x}}{e^{3-x} + e^x}\,dx$. Adding the two forms, $2I = \int_1^2 1\,dx = 1$, so $I = \frac{1}{2}$.

- applies P_3 to get the second form of I (1)
- adds to get $2I = \int_1^2 dx$ (1)
- concludes $I = \frac{1}{2}$ (1)

11. Evaluate $\int_1^{5/2} \sqrt{(8 + 2x - x^2)}\,dx$. [3]

$8 + 2x - x^2 = 9 - (x - 1)^2$. Put $x - 1 = t$; t runs from 0 to $\frac{3}{2}$. $\int_0^{3/2} \sqrt{(9 - t^2)}\,dt = [\frac{t}{2}\sqrt{(9 - t^2)} + \frac{9}{2}\sin^{-1}\frac{t}{3}]_0^{3/2} = \frac{3}{4}\cdot\frac{3\sqrt{3}}{2} + \frac{9}{2}\cdot\frac{\pi}{6} = \frac{9\sqrt{3}}{8} + \frac{3\pi}{4}$.

- completes the square to $9 - (x - 1)^2$ (1)
- uses the formula for $\int \sqrt{(a^2 - t^2)}\,dt$ with the new limits (1)
- value $\frac{9\sqrt{3}}{8} + \frac{3\pi}{4}$ (1)

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