Class 12 Mathematics · Chapter 8: Application of Integrals

Answer key and marking scheme · 20 marks · about 40 minutes · based on NCERT

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1. Given that $\int_0^2 (x-2)\,dx = -2$, the area of the region bounded by $y = x - 2$, the x-axis and the ordinates $x = 0$ and $x = 2$ is [1]

(a) -2 (b) 0
(c) 4 (d) 2

Answer: (d) For $0 < x < 2$ the line lies below the x-axis, so the integral is negative. Only its numerical value is the area: $|-2| = 2$ square units. An area is never negative.

2. If $f(x) < 0$ for every x in [a, b], then the area of the region bounded by $y = f(x)$, the x-axis and the ordinates $x = a$ and $x = b$ is [1]

(a) $\int_a^b f(x)\,dx$ (b) $-\int_a^b f(x)\,dx$
(c) $\int_a^b f(-x)\,dx$ (d) $-\int_b^a f(x)\,dx$

Answer: (b) Below the x-axis every strip has $y < 0$, so $\int_a^b f(x)\,dx$ is negative and the area is its numerical value, $\left|\int_a^b f(x)\,dx\right| = -\int_a^b f(x)\,dx$. $-\int_b^a f(x)\,dx$ equals $\int_a^b f(x)\,dx$, which is negative.

3. The area of the region bounded by $y = e^x$, the x-axis and the ordinates $x = 0$ and $x = \log 3$ is [1]

(a) 3 (b) $\log 3$
(c) 2 (d) $e^3 - 1$

Answer: (c) $A = \int_0^{\log 3} e^x\,dx = e^{\log 3} - e^0 = 3 - 1 = 2$ square units. 3 forgets the lower limit; $e^3 - 1$ takes 3 as the upper limit.

4. The area of one arch of $y = \sin 2x$, between consecutive zeros, is what fraction of the area of one arch of $y = \sin x$? [1]

(a) 2 (b) 1
(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

Answer: (c) One arch of $\sin 2x$ runs from 0 to $\frac{\pi}{2}$: $\int_0^{\pi/2} \sin 2x\,dx = \left[-\frac{1}{2}\cos 2x\right]_0^{\pi/2} = 1$. One arch of $\sin x$ has area $\int_0^{\pi} \sin x\,dx = 2$. The ratio is $\frac{1}{2}$: the arch of $\sin 2x$ has the same height and half the width.

5. The ellipse $\frac{x^2}{a^2} + \frac{y^2}{4} = 1$ encloses an area of 12π square units. Then a equals [1]

(a) 3 (b) 12
(c) 36 (d) 6

Answer: (d) The area enclosed by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab. Here $b = 2$, so $2\pi a = 12\pi$ and $a = 6$. 3 takes b as 4; 12 drops b altogether; 36 is a^2.

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6. Find the area of the region bounded by the curve $y = 4x - x^2$ and the x-axis. [2]

$4x - x^2 = x(4 - x)$ is zero at $x = 0$ and $x = 4$ and positive between. $A = \int_0^4 (4x - x^2)\,dx = [2x^2 - \frac{x^3}{3}]_0^4 = 32 - \frac{64}{3} = \frac{32}{3}$ square units.

- finds the limits $x = 0$ and $x = 4$ and sets up $\int_0^4 (4x - x^2)\,dx$ (1)
- area $\frac{32}{3}$ square units (1)

7. Among all ellipses $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $a + b = 6$, find the one that encloses the greatest area, and that area. [2]

The area enclosed by the ellipse is πab. With $b = 6 - a$, the area is $A = \pi a(6 - a) = \pi[9 - (a - 3)^2]$. This is greatest when $a = 3$ (equivalently $dA/da = \pi(6 - 2a) = 0$, with $d^2A/da^2 = -2\pi < 0$, Chapter 6), so $a = b = 3$: the ellipse is the circle $x^2 + y^2 = 9$, and the greatest area is 9π square units.

- area $\pi a(6 - a)$, from πab with $b = 6 - a$ (1)
- greatest at $a = b = 3$, the circle $x^2 + y^2 = 9$, area 9π square units (1)

8. Find the area of the region enclosed between the curve $y = x^2 - 2x$ and the x-axis. [2]

$x^2 - 2x = x(x - 2)$ is zero at $x = 0$ and $x = 2$ and negative between, so the region lies below the x-axis. $\int_0^2 (x^2 - 2x)\,dx = \frac{8}{3} - 4 = -\frac{4}{3}$; the area is its numerical value, $\frac{4}{3}$ square units.

- limits 0 and 2 and the integral $-\frac{4}{3}$ (1)
- takes the numerical value: area $\frac{4}{3}$ square units (1)

9. Find the area of the region bounded by the curve $y = \sin x + \cos x$, the x-axis and the ordinates $x = 0$ and $x = \pi$. [3]

$\sin x + \cos x = 0$ where $\tan x = -1$, that is at $x = \frac{3\pi}{4}$; the curve is above the x-axis on $(0, \frac{3\pi}{4})$ and below on $(\frac{3\pi}{4}, \pi)$. An anti derivative is $\sin x - \cos x$. $\int_0^{3\pi/4} = (\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}) - (0 - 1) = \sqrt{2} + 1$. $\int_{3\pi/4}^{\pi} = (0 + 1) - \sqrt{2} = 1 - \sqrt{2}$, numerical value $\sqrt{2} - 1$. Area $= (\sqrt{2} + 1) + (\sqrt{2} - 1) = 2\sqrt{2}$ square units.

- zero at $x = \frac{3\pi}{4}$ and the split (1)
- the two integrals $\sqrt{2} + 1$ and $1 - \sqrt{2}$ (1)
- area $2\sqrt{2}$ square units (1)

10. Find the area of the region bounded by the curve $y = x(x - 1)(x - 2)$, the x-axis and the ordinates $x = 0$ and $x = 2$. [3]

$y = x^3 - 3x^2 + 2x$ is positive on (0, 1) and negative on (1, 2). An anti derivative is $F(x) = \frac{x^4}{4} - x^3 + x^2$. $\int_0^1 y\,dx = F(1) - F(0) = \frac{1}{4}$; $\int_1^2 y\,dx = F(2) - F(1) = 0 - \frac{1}{4} = -\frac{1}{4}$. Area $= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ square unit.

- finds the sign change at $x = 1$ and splits the integral (1)
- evaluates the pieces $\frac{1}{4}$ and $-\frac{1}{4}$ (1)
- area $\frac{1}{2}$ square unit (1)

11. Find the area of the region bounded by the curve $y = e^x$, the y-axis and the line $y = e$, (a) using horizontal strips, and (b) as a rectangle less the area under the curve. [3]

The curve meets the y-axis at (0, 1) and the line $y = e$ at (1, e). (a) On the curve $x = \log y$, so $A = \int_1^e \log y\,dy = [y \log y - y]_1^e = (e - e) - (0 - 1) = 1$ square unit. (b) The rectangle with corners (0, 0) and (1, e) has area e; the area under $y = e^x$ from $x = 0$ to $x = 1$ is $e - 1$; the difference is $e - (e - 1) = 1$ square unit.

- limits $y = 1$ to $y = e$ with $x = \log y$ (1)
- integrates $\log y$ by parts to 1 (1)
- rectangle e less $e - 1$ gives 1 (1)

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