Class 12 Mathematics · Chapter 9: Differential Equations

Answer key and marking scheme · 20 marks · about 40 minutes · based on NCERT

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1. If $y = x^m$, $x > 0$, is a solution of $x^2 y'' - 2x y' - 4y = 0$ and $m > 0$, then m equals [1]

(a) 1 (b) 4
(c) 2 (d) 3

Answer: (b) $y' = m x^{m-1}$, $y'' = m(m-1) x^{m-2}$. Substituting: $[m(m-1) - 2m - 4] x^m = 0$, so $m^2 - 3m - 4 = 0$, $(m-4)(m+1) = 0$. The positive root is $m = 4$.

2. The general solution of $\frac{dy}{dx} = x e^{-y}$ is [1]

(a) $y = x^2 e^y + C$ (b) $e^{-y} = \frac{x^2}{2} + C$
(c) $e^y = x^2 + C$ (d) $e^y = \frac{x^2}{2} + C$

Answer: (d) Separating, $e^y\, dy = x\, dx$. Integrating both sides: $e^y = x^2/2 + C$. Option (b) integrates e^y wrongly as e^{-y}; (c) drops the factor 1/2.

3. The sum of the order and the degree of $(\frac{d^3y}{dx^3})^2 - 4(\frac{d^2y}{dx^2})^5 + y = 0$ is [1]

(a) 7 (b) 8
(c) 5 (d) 4

Answer: (c) The highest order derivative is d^3y/dx^3, so the order is 3; its power is 2, so the degree is 2. The sum is 5. Option (b) uses the power 5 of the second derivative as the degree.

4. For which of the following differential equations is the degree not defined? [1]

(a) $y'' + \cos y = 0$ (b) $y' + e^{y'} = x$
(c) $(y')^2 = \sin x$ (d) $y''' + y\, y' = 0$

Answer: (b) $e^{y'}$ is not a polynomial in y', so (b) is not a polynomial equation in its derivatives and its degree is not defined. In (a) cos y involves y, not a derivative, so (a) is a polynomial in y'' of degree 1; (c) has degree 2 and (d) degree 1.

5. The degree of the differential equation $(y'')^2 + (y')^3 - 7y = 0$ is [1]

(a) 1 (b) 3
(c) 2 (d) 5

Answer: (c) The equation is a polynomial in y' and y''. The highest order derivative is y'', and its highest power is 2, so the degree is 2. Option (b) takes the highest power of any derivative; (d) adds the two powers.

6. Find the order and the degree of the differential equation $[1 + (\frac{dy}{dx})^2]^{3/2} = 5\frac{d^2y}{dx^2}$. [2]

The highest order derivative is d^2y/dx^2, so the order is 2. The equation is not yet a polynomial in the derivatives because of the power 3/2; squaring both sides gives $[1 + (dy/dx)^2]^3 = 25 (d^2y/dx^2)^2$, a polynomial equation in y' and y''. The highest power of y'' is 2, so the degree is 2.

- order 2 (1)
- squares to a polynomial in the derivatives and states degree 2 (1)

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7. Find the integrating factor of $\cos x \frac{dy}{dx} + y \sin x = 1$, $0 < x < \frac{\pi}{2}$, and hence its general solution. [2]

Dividing by cos x: $dy/dx + (\tan x)\, y = \sec x$, so $P = \tan x$, $Q = \sec x$. I.F. $= e^{\int \tan x\, dx} = e^{\log \sec x} = \sec x$. Then $y \sec x = \int \sec^2 x\, dx = \tan x + C$, so $y = \sin x + C \cos x$.

- standard form and I.F. = sec x (1)
- $y = \sin x + C \cos x$ (1)

8. A colony of bacteria grows at a rate proportional to the number N present, and it trebles in 4 hours. [2] Write the differential equation for N and show that $N(t + 4) = 3N(t)$ for every t, so the trebling time does not depend on the number at the start.

$dN/dt = kN$. Separating, $dN/N = k\, dt$, so $N = N_0 e^{kt}$, N_0 the starting number; trebling in 4 hours gives $e^{4k} = 3$. Then $N(t + 4) = N_0 e^{kt} e^{4k} = 3 N_0 e^{kt} = 3N(t)$ for every t, whatever N_0 is.

- $dN/dt = kN$, $N = N_0 e^{kt}$ and $e^{4k} = 3$ (1)
- $N(t + 4) = N_0 e^{kt} e^{4k} = 3N(t)$, independent of N_0 (1)

9. Find the general solution of $\frac{dy}{dx} + 4y = 8x$. [3]

$P = 4$, $Q = 8x$. I.F. $= e^{4x}$. Then $y e^{4x} = \int 8x e^{4x}\, dx = 8[x e^{4x}/4 - e^{4x}/16] + C = 2x e^{4x} - e^{4x}/2 + C$ (integration by parts). Dividing by e^{4x}: $y = 2x - 1/2 + C e^{-4x}$.

- I.F. $= e^{4x}$ (1)
- $\int 8x e^{4x}\, dx = 2x e^{4x} - \frac{1}{2} e^{4x}$ by parts (1)
- $y = 2x - \frac{1}{2} + C e^{-4x}$ (1)

10. Find the particular solution of $(1 + x^2) \frac{dy}{dx} = xy$, $y > 0$, given that $y = 2$ when $x = 0$. [3]

Separating, $dy/y = x\, dx/(1 + x^2)$. Integrating, $\log y = (1/2) \log (1 + x^2) + \log C$, so $y = C\sqrt{(1 + x^2)}$. At $x = 0$, $y = 2$: $C = 2$. The particular solution is $y = 2\sqrt{(1 + x^2)}$.

- separates the variables (1)
- integrates: $y = C\sqrt{(1 + x^2)}$ (1)
- $C = 2$ and $y = 2\sqrt{(1 + x^2)}$ (1)

11. Solve $x \frac{dy}{dx} - 3y = x^4 \cos x$, $x > 0$. [3]

Dividing by x: $dy/dx - (3/x)\, y = x^3 \cos x$, so $P = -3/x$. I.F. $= e^{-3 \log x} = e^{\log x^{-3}} = 1/x^3$. Then $y/x^3 = \int x^3 \cos x \cdot (1/x^3)\, dx = \int \cos x\, dx = \sin x + C$. Hence $y = x^3 \sin x + C x^3$.

- standard form with $P = -\frac{3}{x}$ (1)
- I.F. $= \frac{1}{x^3}$ (1)
- $y = x^3 \sin x + C x^3$ (1)